DESIGN OF AN IMPROVED PERFORMANCE DUAL-BAND POWER DIVIDER

Stelios Tsitsos, Anastasios Papatsoris, Ioanna Peikou, and Athina Hatziapostolou

Department of Computer Engineering, Communications and Networks Group, Technological and Educational Institute (TEI) of Central Macedonia, Magnesias End, GR-62124, Serres, Greece

Abstract

In this paper an improved performance dual-band power divider is presented with respect to output return loss and port isolation. The proposed circuit features a transmission line only structure (plus the isolation resistor) thus avoiding the parasitic effects of lumped components. Additionally it can be easily implemented since it employs realistic characteristic impedance values. Analytical expressions for the design equations are derived using the even and odd mode analysis.

Keywords: Power divider; dual-band; distributed structure.

1. INTRODUCTION

Power dividers/combiners are key elements in modern RF front-end communication systems. They are used in various applications such as power division and/or combination in antenna arrays distribution networks, microwave mixers, amplifiers and oscillators, as well as in high speed digital integrated circuits.

In recent years the advances in wireless and mobile communications require a dual-band or multiband operation (i.e. WLANs, GSM, UMTS etc.). To satisfy this requirement several dual-band or multi-band power dividers have been proposed employing transmission lines and/or lumped elements [1]-[7]. One interesting dual-band design which employs distributed structures only (plus the isolation resistor) [4] achieves a compact design with reduced losses using realistic impedance values for the implementation of the transmission lines (Fig. 1). Although a high level of input return loss and port isolation can be achieved, the output return loss and the port isolation are not ideal in the region between the two frequencies of interest. For example, S22 does not achieve the ideal 0 dB value in the region between the two frequencies of interest (see Fig. 2). Although this is not important in power splitting operation it may be particularly important in the power combining operation.





Figure 1: Conventional dual-band power divider [4].

Figure 2: Simulated: (a) Input return loss. (b) Output return loss. (c) Insertion loss ($|S_{31}|$ is identical to $|S_{21}|$, due to symmetry). (d) Port isolation, for different values of the isolation resistor [4].

In this work an improved performance dual-band power divider is presented in Fig. 3. This involves additional transmission lines and open-circuited stubs to achieve better output return loss and port isolation results. In addition the proposed structure uses practical characteristic impedance values that are easily implemented.



Figure 3: Proposed dual-band power divider.

2. CIRCUIT ANALYSIS

The proposed dual-band power divider presented in Fig. 3 features four $\lambda/4$ branch transmission lines with characteristic impedances Z_A and Z_B , two $\lambda/4$ series transmission lines of characteristic impedance Z_D , three $\lambda/4$ open-circuited shunt stubs of characteristic impedances Z_C and Z_E and an isolation resistor R. This circuit may be theoretically analysed with the aid of the even and odd mode analysis. Thus the circuit of Fig. 3 is replaced by the half-circuits of the even and odd mode configuration (Figs. 4(a) and 4(b)).





Figure 4: Half circuit of the proposed power divider: (a) Even mode. (b) Odd mode.

2.1.1 Even-Mode Analysis

The even-mode half circuit of Fig. 4(a) consists of two serial branch-lines with characteristic impedances Z_A and Z_D respectively and three shunt elements with characteristic admittances Y_1 , Y_2 and Y_3 respectively, given as follows:

$$Y_1 = \frac{1}{2} Y_C \tan\theta \tag{1}$$

$$Y_2 = Y_B \tan \theta \tag{2}$$

$$Y_3 = Y_E \tan\theta \tag{3}$$

The ABCD matrix representation of the half-circuit of Fig. 4(a) may be derived as [4], [8]:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ jY_1 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & jZ_A \sin\theta \\ jY_A \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ jY_2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ jY_3 & 1 \end{bmatrix}$$
(4)

where:

$$\theta = \frac{\pi}{2} \frac{f}{f_0} \tag{5}$$

and:

$$f_0 = \frac{f_1 + f_2}{2}$$
(6)

From (4) the following equations may be derived:

$A = \cos^2\theta - Y_2 Z_A \sin\theta \cos\theta - Y_D Z_A \sin^2\theta - Y_3 Z_D \sin\theta \cos\theta + Y_2 Y_3 Z_A Z_D \sin^2\theta -$	
Y ₃ Z _A sinθcosθ	(7)

$$B = jZ_{D}\sin\theta\cos\theta - jY_{2}Z_{A}Z_{D}\sin^{2}\theta + jZ_{A}\sin\theta\cos\theta$$
(8)

$$C = jY_{1}\cos^{2}\theta + jY_{A}\sin\theta\cos\theta - jY_{1}Y_{2}Z_{A}\sin\theta\cos\theta + jY_{2}\cos^{2}\theta - jY_{1}Y_{D}Z_{A}\sin^{2}\theta + jY_{D}\sin\theta\cos\theta - jY_{1}Y_{3}Z_{D}\sin\theta\cos\theta - jY_{3}Y_{A}Z_{D}\sin^{2}\theta + jY_{1}Y_{2}Y_{3}Z_{A}Z_{D}\sin^{2}\theta - jY_{2}Y_{3}Z_{D}\sin\theta\cos\theta - jY_{1}Y_{3}Z_{A}\sin\theta\cos\theta + jY_{3}\cos^{2}\theta$$
(9)

$$D = -Y_1 Z_D \sin\theta \cos\theta - Y_A Z_D \sin^2\theta + Y_1 Y_2 Z_A Z_D \sin^2\theta - Y_2 Z_D \sin\theta \cos\theta - Y_1 Z_A \sin\theta \cos\theta + \cos^2\theta$$
(10)

The input impedance of the half-circuit of Fig. 4(a) may be expressed as follows:

$$Z_{in} = \frac{AZ_0 + B}{CZ_0 + D} = 2Z_0$$
(11)

Assuming that the network is reciprocal and lossless, A and D are real quantities, while B and C are imaginary quantities. Then:

$$A = 2D = \pm \sqrt{2 + B^2 Y_0^2}$$
(12)

Setting:

$$k_1 = \frac{Z_A}{Z_B} \qquad \qquad k_2 = \frac{Z_A}{Z_D} \qquad \qquad k_3 = \frac{Z_A}{Z_E} \tag{13}$$

and using equations (1), (2), (3), (7), (10) and also A=2D from (12) the following expression for Z_C is obtained, after some mathematical manipulations:

$$z_{c} \equiv \frac{(1-k_{1}\tan^{2}\theta+k_{2})Z_{A}}{k_{2}\cot^{2}\theta-2-2k_{1}-k_{1}k_{3}\tan^{2}\theta+k_{1}k_{2}+k_{3}+k_{2}k_{3}+k_{2}^{2}}$$
(14)

Additionally using equations (1), (2), (3), (7), (8) and also $A = \pm \sqrt{2 + B^2 Y_0^2}$ from (12) the following expression is obtained, after some mathematical manipulations:

$$\frac{2k_{2}^{2}}{\sin^{4}\theta} - k_{2}^{2}\cot^{4}\theta - k_{1}^{2}k_{3}^{2}\tan^{4}\theta + 2k_{2}(k_{1}k_{2} + k_{3} + k_{3}k_{2} + k_{2}^{2})\cot^{2}\theta + 2k_{1}(k_{3}k_{2}^{2} + k_{1}k_{3}k_{2} + k_{3}^{2} + k_{3}^{2}k_{2})\tan^{2}\theta - k_{2}^{2}(k_{1} + k_{2} + k_{3})^{2} - 2k_{2}^{2}k_{3} Z_{A}^{2}Y_{0}^{2} = \frac{-4k_{1}k_{2}k_{3} - k_{3}^{2} - 2k_{3}^{2}k_{2}}{k_{2}^{2}\cot^{2}\theta + 2k_{2}\cot^{2}\theta + \cot^{2}\theta + k_{1}^{2}\tan^{2}\theta - 2k_{1}k_{2} - 2k_{1}}$$
(15)

2.1.2 Odd-Mode Analysis

The odd-mode half circuit is presented in Fig. 4(b). The admittances $Y_{out(1)}$ and $Y_{out(2)}$ are derived as follows:

$$Y_{out(1)} = Y_{B} \frac{G + jY_{B} \tan\theta}{Y_{B} + jG \tan\theta} - jY_{A} \cot\theta = \frac{Y_{B}G + jY_{B}^{2} \tan\theta - jY_{A}Y_{B} \cot\theta + Y_{A}G}{Y_{B} + jG \tan\theta}$$
(16)

$$Y_{out(2)} = Y_{D} \frac{Y_{out(1)} + jY_{D} \tan\theta}{Y_{D} + jY_{out(1)} \tan\theta}$$
(17)

The output admittance Y_{out} is calculated as follows:

$$Y_{out} = Y_0 = Y_{out(2)} + jY_E \tan\theta$$
(18)

After some mathematical manipulations the following expression is derived:

$$Y_{out} = \frac{Y_B Y_D G + jY_B^2 Y_D \tan \theta - jY_A Y_B Y_D \cot \theta + Y_A Y_D G + jY_B Y_D^2 \tan \theta - GY_D^2 \tan^2 \theta + jY_B Y_D Y_E \tan \theta - GY_D Y_E \tan^2 \theta - Y_B Y_E G \tan^2 \theta - jY_B^2 Y_E \tan^2 \theta + jY_A Y_B Y_E \tan \theta}{Y_B Y_D + jY_D G \tan \theta + jY_B G \tan \theta - Y_B^2 \tan^2 \theta + Y_A Y_B + jY_A G \tan \theta}$$
(19)

By equating real and imaginary parts the following expressions are obtained:

$$G = \frac{Y_0(k_1 + k_1k_2 - k_1^2\tan^2\theta)}{k_2 + k_1k_2 + (-k_3 - k_1k_3 - k_2k_3 - k_2^2)\tan^2\theta}$$
(20)

$$Z_{A}^{2}Y_{0}^{2} = \frac{\left(k_{2}^{2}+k_{3}+k_{1}k_{2}+k_{2}k_{3}-k_{2}\text{cot}^{2}\theta-k_{1}k_{3}\tan^{2}\theta\right)\left(k_{2}+k_{1}k_{2}+\left(-k_{3}-k_{1}k_{3}-k_{2}k_{3}-k_{2}^{2}\right)\tan^{2}\theta\right)}{1+k_{1}+2k_{2}+k_{2}^{2}+k_{1}k_{2}-k_{1}(1+k_{1}+k_{2})\tan^{2}\theta} (21)$$

2.1.3 Design equations Combining equations (15) and (21) the following design equations are derived:

$$k_{3=} \frac{\begin{bmatrix} \left(-2k_{2}^{2}-2k_{2}^{2}k_{1}-4k_{2}^{3}-2k_{2}^{4}-2k_{2}^{3}k_{1}\right)\frac{1}{\sin^{4}\theta}+\left(+2k_{2}^{2}k_{1}+2k_{2}^{2}k_{1}^{2}+2k_{2}^{3}k_{1}\right)\frac{1}{\sin^{2}\theta\cos^{2}\theta}+\left(-k_{1}k_{2}^{3}-k_{1}k_{2}^{4}\right)\cot^{4}\theta}{+\left(-k_{1}^{2}k_{2}^{4}-k_{1}^{3}k_{2}^{3}\right)\tan^{4}\theta+\left(-2k_{1}k_{2}^{3}-k_{1}k_{2}^{4}+2k_{1}^{2}k_{2}^{3}+k_{1}k_{2}^{5}+k_{1}^{2}k_{2}^{4}\right)\cot^{2}\theta}{+\left(+2k_{1}^{2}k_{2}^{3}+k_{1}k_{2}^{4}-2k_{1}^{3}k_{2}^{3}-k_{1}^{2}k_{2}^{4}+k_{1}k_{2}^{5}\right)\tan^{2}\theta+\left(-k_{1}k_{2}^{3}+4k_{1}^{2}k_{2}^{3}+k_{1}k_{2}^{4}-k_{1}^{3}k_{2}^{3}+k_{2}^{4}k_{1}^{2}+2k_{1}k_{2}^{5}\right)}\right]}\left(\frac{-k_{1}^{3}k_{2}^{2}\tan^{6}\theta+2k_{1}^{2}k_{2}^{2}\tan^{4}\theta+2k_{1}^{2}k_{2}^{3}\tan^{4}\theta-2k_{1}^{3}k_{2}^{2}\tan^{4}\theta-k_{1}k_{2}^{2}\cot^{2}\theta-2k_{1}k_{2}^{3}\cot^{2}\theta}{\left(-k_{1}k_{2}^{4}\cot^{2}\theta-k_{1}k_{2}^{2}\tan^{2}\theta+4k_{1}^{2}k_{2}^{2}\tan^{2}\theta-2k_{1}k_{2}^{3}\tan^{2}\theta-k_{1}k_{2}^{4}\tan^{2}\theta}{+4k_{1}^{2}k_{2}^{2}\tan^{2}\theta-2k_{1}k_{2}^{2}+2k_{1}^{2}k_{2}^{2}-2k_{1}k_{2}^{3}-2k_{1}k_{2}^{4}}\right)}\right)}$$

$$(22)$$

$$\begin{split} \mathbf{z}_{A} \begin{pmatrix} +k_{1}^{4}k_{2}^{2}tan^{6}\theta - 3k_{1}^{2}k_{2}^{2}tan^{6}\theta - 3k_{1}^{2}k_{2}^{2}tan^{6}\theta - 3k_{1}^{2}k_{2}^{2}tan^{6}\theta - 3k_{1}^{2}k_{2}^{2}tan^{4}\theta + 3k_{1}^{2}k_{2}^{2}tan^{6}\theta - 6k_{1}^{2}k_{2}^{2}tan^{4}\theta + k_{2}^{4}k_{2}^{2}tan^{4}\theta - k_{1}^{4}k_{2}^{2}tan^{4}\theta - k_{1}^{4}k_{2}^{4}tan^{4}\theta - k_{1}^{4}k_{2}^{4}tan^{4}\theta - k_{1}^{4}k_{2}^{4}tan^{4}\theta - k_{1}^{4}k_{2}^{4}tan^{4}\theta - k_{1}^{4}k_{2}^{4}tan^{4}\theta + k_{1}^{4}k_{2}^{4}tan^{4}\theta + 2k_{1}^{4}k_{2}^{4}tan^{4}\theta - k_{1}^{4}k_{2}^{4}tan^{4}\theta - 4k_{1}^{4}k_{2}^{4}tan^{4}\theta + 2k_{1}^{4}k_{2}^{4}tan^{4}\theta - k_{1}^{4}k_{2}^{4}tan^{4}\theta - k_{1}^{4}k_{2}^{4}tan^{4}\theta - k_{1}^{4}k_{2}^{4}tan^{4}\theta + k_{1}^{4}k_{2}^{4}tan^{4}\theta - 2k_{1}^{4}k_{2}^{4}tan^{4}\theta - 2k_{1}^{4}k_{2}^{4}tan^{2}\theta - 2k_{1}^{4}k_{2}^{4}tan^{2}\theta - 2k_{1}^{4}k_{2}^{4}tan^{2}\theta - 2k_{1}^{4}k_{2}^{4}tan^{2}\theta - 2k_{1}^{4}k_{2}^{4}tan^{2}\theta + 2k_{1}^{4}k_{2}^{2}dk^{2} + k_{1}^{4}k_{2}^{2}dk^{2} + k_{1}^{4}k_{2}^{2}dk^{2}}dk^{2} + k_{1}^{4}k_{2}^{2}dk^{2}$$

$$Z_{A} = \begin{cases} \left(\begin{array}{c} -\frac{2k_{2}^{2}}{\sin^{4}\theta} - \frac{2k_{2}^{2}k_{1}}{\sin^{4}\theta} - \frac{6k_{2}^{2}}{\sin^{4}\theta} \\ -\frac{6k_{2}^{4}}{\sin^{4}\theta} - \frac{4k_{2}^{2}k_{1}}{\sin^{4}\theta} - \frac{2k_{2}^{5}}{\sin^{4}\theta} \\ -\frac{6k_{2}^{4}}{\sin^{4}\theta} - \frac{4k_{2}^{2}k_{1}}{\sin^{4}\theta} - \frac{2k_{2}^{5}}{\sin^{4}\theta} \\ -\frac{2k_{2}^{2}k_{1}}{\sin^{4}\theta} + \frac{4k_{2}^{2}k_{1}}{\sin^{2}\theta\cos^{2}\theta} + \frac{4k_{2}^{2}k_{1}}{\sin^{2}\theta\cos^{2}\theta} \\ +\frac{2k_{2}^{2}k_{1}}{\sin^{4}\theta} + \frac{4k_{2}^{2}k_{1}}{\sin^{2}\theta\cos^{2}\theta} + \frac{4k_{2}^{2}k_{1}}{\sin^{2}\theta\cos^{2}\theta} \\ -\frac{2k_{2}^{2}k_{2}}{\sin^{4}\theta} - \frac{4k_{2}^{2}k_{1}}{\sin^{2}\theta\cos^{2}\theta} + \frac{4k_{2}^{2}k_{1}}{\sin^{2}\theta\cos^{2}\theta} \\ -\frac{2k_{1}^{2}k_{2}}{\cos^{4}\theta} - \frac{2k_{2}^{2}k_{1}^{2}}{\cos^{4}\theta} - \frac{2k_{1}^{2}k_{2}^{3}}{\cos^{4}\theta} \\ -\frac{2k_{1}^{2}k_{2}}{\cos^{4}\theta} - \frac{2k_{2}^{2}k_{1}^{2}}{\cos^{4}\theta} - \frac{2k_{1}^{2}k_{2}^{3}}{\cos^{4}\theta} \\ -\frac{2k_{1}^{2}k_{2}}{\cos^{4}\theta} - \frac{2k_{2}^{2}k_{1}^{3}}{\cos^{4}\theta} - \frac{2k_{1}^{2}k_{2}^{3}}{\cos^{4}\theta} \\ -\frac{2k_{1}^{2}k_{2}^{3}}{\cos^{4}\theta} - k_{1}^{2}k_{2}^{3}ta^{2}\theta} \\ -\frac{2k_{1}^{2}k_{2}^{2}}{\cos^{4}\theta} - \frac{2k_{1}^{2}k_{2}^{3}}{\cos^{4}\theta} \\ -\frac{2k_{1}^{2}k_{2}^{3}}{\cos^{4}\theta} - k_{1}^{2}k_{2}^{3}ta^{2}\theta} \\ -\frac{2k_{1}^{2}k_{2}^{3}}{\cos^{4}\theta} - k_{1}^{2}k_{2}^{3}ta^{2}\theta} \\ -\frac{2k_{1}^{2}k_{2}^{3}}{\cos^{4}\theta} - k_{1}^{2}k_{2}^{3}ta^{2}\theta} \\ -k_{1}^{2}k_{2}^{3}ta^{4}\theta} - k_{1}^{2}k_{2}^{3}ta^{4}\theta} \\ -k_{1}^{2}k_{2}^{3}ta^{4}\theta} - k_{1}^{2}k_{2}^{3}ta^{4}\theta} \\ -k_{1}^{2}k_{2}^{3}ta^{4}\theta} - k_{1}^{2}k_{2}^{3}ta^{4}\theta} \\ +\frac{k_{1}^{2}k_{2}^{3}ta^{4}\theta} - 3k_{1}^{3}k_{2}^{3}ta^{4}\theta} \\ +\frac{k_{1}^{2}k_{2}^{3}ta^{4}\theta} - 3k_{1}^{3}k_{2}^{3}ta^{4}\theta} \\ +\frac{k_{1}^{2}k_{2}^{3}ta^{4}\theta} - 3k_{1}^{3}k_{2}^{3}ta^{4}\theta} \\ +\frac{k_{1}^{2}k_{2}^{3}ta^{4}\theta} \\ +\frac{k_{1}^{2}k_{2}^{3}ta^{4}\theta} - 3k_{1}^{3}k_{2}^{3}ta^{4}\theta} \\ +\frac{k_{1}^{2}k_{2}^{3}ta^{4}\theta} \\ +\frac{k_{1}^{2}$$

From equations (23), (24) and (25) the parameters Z_A , Z_C and R of the proposed dual-band power divider (Fig. 3) may be calculated. Subsequently by selecting appropriate values for k_1 and k_2 , k_3 may be calculated from (22) and using (13) the parameters Z_B , Z_D and Z_E may also be calculated.

3. IMPLEMENTATION AND RESULTS

The design equations derived previously are satisfied for a number of solutions. One possible solution which results in realisable impedance values is given in the following Table.

Z _A (Ohms)	Z _B (Ohms)	Z _C (Ohms)	Z _D (Ohms)	Z _E (Ohms)	R (Ohms)
65.45	20.10	59.34	25.95	29.68	297.74

Table I: Impedance and isolation resistor values for the circuit of Fig. 3.

Subsequently the proposed divider illustrated in Fig. 3 was simulated in the Advanced Design SystemTM [9] and its response is presented in Fig. 5.





By comparing the responses in Figs. 2 and 5, it is clear that the proposed circuit of Fig. 2 offers an improved performance especially with respect to the output return loss (S22 or S33) and the isolation (S32 or S23). For example S22 achieves the ideal 0 dB value in the region between the two frequencies of interest and port isolation exhibits very high values in the same region.

To verify the predicted results, the proposed dual-band power divider of Fig. 3 was constructed on a PCB using a RT 5880 substrate with a dielectric constant of 2.2 (Fig. 6). Three 100 Ohms microwave resistors were used to achieve the necessary isolation between output ports. For increased accuracy the layout was fined tuned by an electromagnetic simulator to take into account the effect of junction

discontinuities. The scattering parameters of the PCB were measured using an Agilent E5071C VNA over the frequency range 0.5 - 2.5 GHz. The comparison between measured and electromagnetic simulation results is presented in Fig. 7. Good agreement is observed and the discrepancies are mainly due to the limited accuracy of the etching process used.



Figure 6: Fabricated dual-band power divider.



Figure 5: Measured and electromagnetic simulation results for: (a) Input return loss. (b), (c) Output return loss. (d), (e) Insertion loss. (f) Port isolation, for the proposed dual-band divider of Fig. 3.

3. CONCLUSIONS

In this work an improved performance dual-band power divider was designed and simulated. The mathematical analysis and the extraction of the design equations were based on the even and odd mode analysis. Simulated and measured results indicated an improved output return loss performance as well as improved port isolation.

4. ACKNOWLEDGEMENTS

The authors would like to acknowledge that this research work has been co-financed by the European Union (European Social Fund – ESF) and Greek national funds through the Operational Program "Education and Lifelong Learning" of the National Strategic Reference Framework (NSRF) - Research Funding Program: **ARCHIMEDES III** – Investing in knowledge society through the European Social Fund.

The authors would like to thank Dr. S. Lucyszyn and Mr. F. Hu, for providing the microwave resistors for the fabrication of the PCB and also Mr. D. Manos and Mr. D. Valalas for fabricating the PCB.

5. **REFERENCES**

- [1] L. Wu, Z. Sun, H. Yilmaz, and M. Berroth, "A dual-frequency Wilkinson power divider", *IEEE Transactions on Microwave Theory and Techniques*, Vol. 54, No. 1, pp. 278–284, Jan. 2006.
- [2] K.-K.M. Cheng, and F.-L. Wong, "A new Wilkinson power divider design for dual-band application", *IEEE Microwave and Wireless Components Letters*, Vol. 17, No. 9, pp. 664–666, Sept. 2007.
- [3] S.-H. Ahn, J.W. Lee, C.S. Cho, and T.K. Lee, "A Wilkinson power divider with different power ratios at different frequencies", *APMC 2007 Asia-Pacific Microwave Conference*, pp. 1-4, 11-14 Dec. 2007.
- K.-K.M. Cheng, and C. Law, "A novel approach to the design and implementation of dualband power divider", *IEEE Transactions on Microwave Theory and Techniques*", Vol. 56, No. 2, pp. 487–492, Feb. 2008.
- [5] Y. Wu, Y. Liu, Y. Zhang, J. Gao, and H. Zhou, "A dual-band unequal Wilkinson power divider without reactive components", *IEEE Transactions on Microwave Theory and Techniques*", Vol. 57, No. 1, pp. 216-222, Jan. 2009.
- [6] Z. Wang, J. S. Jang, and C.-W. Park, "Compact dual-band Wilkinson power divider using lumped component resonators and open-circuited stubs, *IEEE 12th Annual Wireless and Microwave Technology Conference (WAMICON)*, pp. 1-4, 2011.
- [7] Q.-Xin Chu, F. Lin, Z. L. Z. Gong, "Novel Design Method of Tri-Band Power Divider", *IEEE Transactions on Microwave Theory and Techniques*", Vol. 59, No. 9, pp. 2221-2226, Sept. 2011.
- [8] D.M. Pozar, *Microwave Engineering*, N.York: Wiley, 2005, Chapter 7.
- [9] Advanced Design SystemTM, *Agilent Technologies*, 2008 (update 1).