RE-ASSESSMENT OF THE APPLICATION OF THE 45^0 HEAT SPREADING ANGLE RULE FOR RF POWER TRANSISTORS

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This paper compares the thermal resistance of RF power transistors calculated using the 45^0 heat spreading rule with exact analytic values. It is shown that the 45^0 rule can under-estimate the thermal resistance by up to 40% for transistors with thin flanges and in which the die are small in comparison with the lateral dimensions of the package.

INTRODUCTION

The 45^0 heat-spreading approximation as a means of calculating thermal resistance was first introduced by Holway and Adlerstein [1] in 1977. They were concerned with calculating the thermal resistance of IMPATT diodes. In 1977 there were no commercially-available general-purpose thermal analysis programs, and neither were there any personal computers; the only tool available to the average engineer was a pocket calculator. Consequently, there was a real need for a simple method of calculating the thermal resistance since the only alternative was to program a mainframe computer which was not only very time-consuming but engineers working in smaller companies didn't even have access to this resource. Figure 1 shows the geometry of the problem that they were considering with their results.

Examining Figure 1 suggests that the maximum error is no more than 12.5% between the exact and approximate values of thermal resistance. However, there are certain issues with this analysis. Firstly, their analysis indicates that the approximate method always underestimates the thermal resistance except when the diameter of the diode is the same as the gold heatsink when it overestimates it by 10%. No explanation was given for this reversal. Their reference value for thermal resistance was a 2D numerical analysis which they assumed was giving the exact value, but when the diode diameter is the same as the gold heatsink then no heat spreading occurs, the heat flow is purely columnar and the thermal resistance can be calculated exactly very easily. The approximate and numerical values for thermal resistance should exactly agree in this case but they don't, there being a 10% difference. A simple hand calculation shows that the problem lies with the numerical analysis and not the approximate calculation which is exact when the diode diameter equals the heatsink diameter. Consequently, there is some doubt over how accurate the numerical analysis is for all other values of diode diameter.

Another issue is that Holway and Adlerstein were not calculating the thermal resistance of the IMPATT diode by itself but of the diode mounted in its heatsink which they assumed to be a semi-infinite copper block. When the diode diameter in Figure 1 is 6 mils, for example, the semi-infinite copper block contributes about one-third of the total thermal resistance. If the semi-infinite copper block is removed to allow for the calculation of the thermal resistance of the diode by itself then the error using the approximate calculation may be a lot larger than the 12.5% shown in Figure 1.

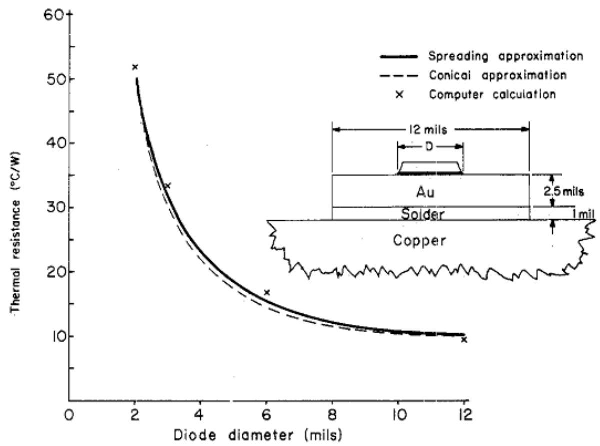

Figure 1. The structure analysed by Holway & Adlerstein [1]

Figure 2 shows a typical Gunn or IMPATT diode oscillator. These oscillators dissipate typically a few Watts as heat and so the large metal block that houses the diode forms an adequate heatsink even if left in ambient air. Consequently, Holway and Adlerstein's assumption of a semi-infinite heatsink was reasonable. However, today's RF power transistors can dissipate hundreds of Watts and they would fail almost instantly if left in ambient air. RF power transistors need to be attached to a base-plate that maintains the underneath side of the transistor at a constant temperature. This change of boundary condition from a semi-infinite heatsink to a constant temperature also has an extremely fortunate consequence, namely that an exact analytic calculation of the temperature anywhere within the structure is possible and so there is no need to resort to a numerical analysis. Since an exact calculation is possible, then why bother using the approximate spreading calculation? The reason is that the exact calculation requires the summation of an infinite series whereas the approximate calculation is quick and easy requiring only a pocket calculator. The approximate method is still widely used, not just to calculate the thermal resistance of a complete packaged transistor but also to determine the appropriate spacing between the individual die in the package as well as to determine the gate to gate spacing on the die etc., but the question is how accurate is the answer? Given the uncertainties associated with Holway and Adlerstein's analysis it seems appropriate to re-examine the accuracy with specific reference to its application to RF power transistors.

Figure 2. Typical Gunn or IMPATT oscillator. (Photo courtesy of ZAX).

THERMAL ANALYSIS OF AN RF POWER TRANSISTOR

Figure 3 shows a typical RF power transistor. One or more rectangular die are soldered to a large rectangular metal flange. It is possible to calculate the temperature at any point in this structure analytically for either a single heat-generating area on a multiple-layered heatsink, or for multiple heat-generating areas on a single-layered heatsink [2]. However, the temperature depends on x, y and z. It will be shown that the error between the exact and approximate solutions depends only on the ratio of two dimensions and so it is sufficient to consider a simple 2D circular geometry on a multiple-layered heatsink. Figure 4 shows the structure to be analysed.

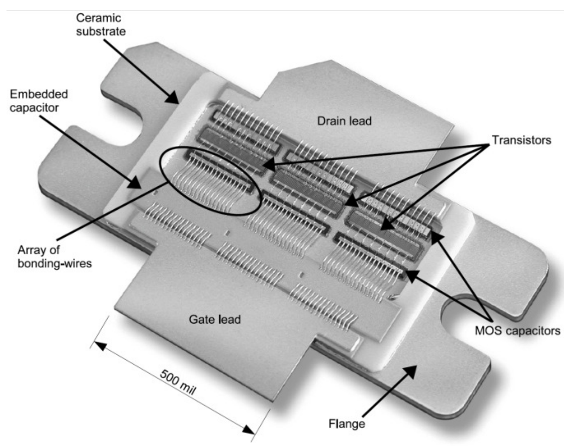

Figure 3. Construction of RF Power Transistor. (Photo courtesy Cambridge University Press).

In the analysis that follows it is assumed that there is zero heat loss by either convection or radiation and that all heat loss is entirely by conduction. The boundary conditions for the problem are:

$$T_n(r, l_n) = 0 \text{ for } 0 < r < B \quad (1)$$

$$\frac{\partial T_i}{\partial r} = 0 \text{ for } r = B,\ 0 < z_i < l_i \quad (2)$$

$$\frac{\partial T_1}{\partial z_1} = \begin{matrix} -\frac{f}{\sigma_1} \ \ for \ \ 0 < r < A \\ 0 \ \ for \ A < r < B \end{matrix} \qquad (3)$$

where *f* is the heat flux density dissipated uniformly over a circular area of radius *A* and *T* is the temperature. Apart from the bottom surface and the area where the heat flux enters, all other surfaces are adiabatic as defined by equations (2) and (3).

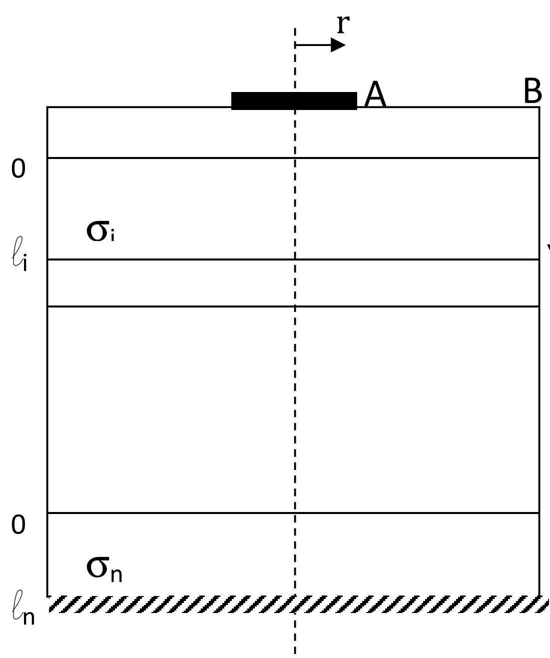

Figure 4. Structure for analysis.

Brook and Smith [3,4] determined an exact analytic solution for the temperature at any point within this structure. The highest temperature occurs at the top centre of the structure i.e. at *r* = *z* = 0 and, in the case of just one layer, is given by:

$$T = \frac{f}{\sigma}\frac{A^2}{B^2} l + \frac{2fA}{\sigma}\sum_{j=1}^{\infty} \tanh\left(\alpha_j \frac{l}{B}\right) \frac{J_1\left(\alpha_j \frac{A}{B}\right)}{\alpha_j^2 J_0^2(\alpha_j)} \qquad (4)$$

where J_0 and J_1 are the first and second order Bessel functions, respectively, and α_j is the *j*th order zero of J_1 and σ is the thermal conductivity. Bessel functions are one of the standard functions within Excel™ so that equation 4 is easily evaluated using a spreadsheet calculation. It can be immediately deduced from the expression given by Brook and Smith for the temperature at any point in the structure shown in Figure 4 that the value of σ has no effect on the temperature profile, a change in the value of σ simply scales the temperature at all points uniformly.

The thermal resistance R_{th} is given by $Rth = T/(f\pi A^2)$ and this is plotted in a normalized form of $\sigma\pi AR_{th}$ in Figure 5. The normalized thermal resistance depends only on the two ratios l/B and A/B and so Figure 5 is a set of universal design curves. For a sufficiently large thickness then the heat flow adjacent to the bottom is purely columnar so that eventually all the curves become straight lines, as expected, regardless of the value of A/B.

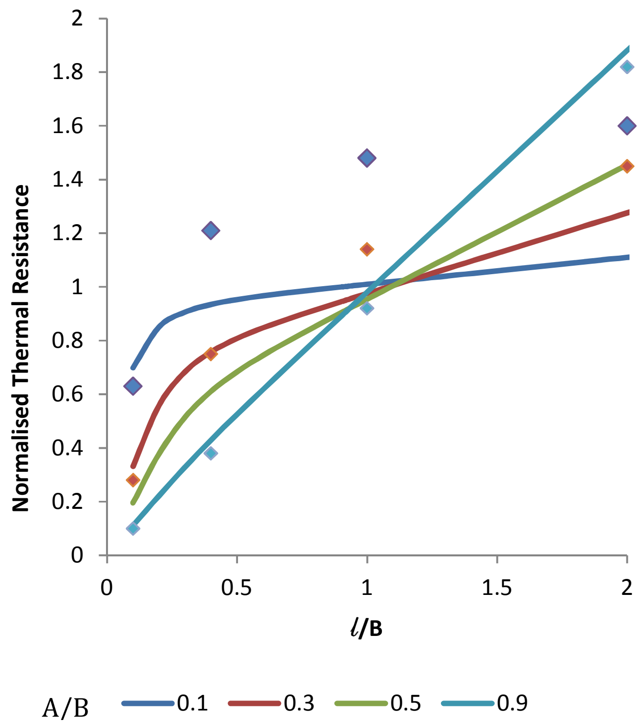

Figure 5. Graph of Normalised Thermal Resistance vs thickness with heat flux radius as parameter. The individual data points are the values of thermal resistance as determined by the 45^0 heat-spreading approximation.

The 45^0 heat spreading solution for Figure 4 is obtained by assuming that the heat flux spreads out in a 45^0 cone as shown in Figure 6, and that the heat flux is uniform at every cross-section of the cone i.e. for all values of z. Once the heat flux lines reach the outside radius then pure columnar heat flow occurs. Elementary analysis shows that the temperature at $z = 0$ for an arbitrary value of θ is given by:

$$T = \frac{fA}{\sigma} \frac{l/A}{1+\frac{l}{A}\tan\theta} \qquad (5)$$

Equation (5) corresponds to equation (4) in the exact analysis. Holway and Adlerstein argued that in the limit as $l \rightarrow \infty$ then equation (4) must reduce to the well-known expression for the thermal resistance of a circular disk transmitting a uniform heat flux into a semi-infinite medium [5] which is given by

$$R_{th} = \frac{1}{\pi\sigma A} \qquad (6)$$

This requires that $\theta = 45^0$ and is the origin of the 45^0 rule. The normalized version of Equation (5) is also plotted in Figure 5 as a series of distinct data points. It can be seen that the 45^0 rule always underestimates the peak temperature and that the error can be as high as 40%. The error is greatest for structures in which the heat-flow is mainly by spreading with little columnar heat flow i.e. for thin layers with die small in area by comparison with the heat-sink size.

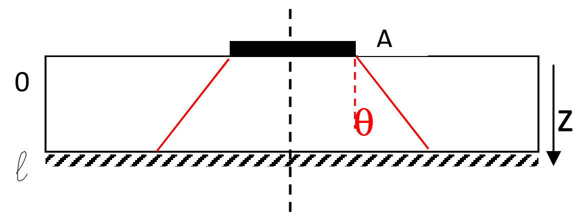

Figure 6. Heat flux spreading out at angle θ.

DOES A SMALLER HEAT SPREADING ANGLE REDUCE THE ERROR?

Since the 45^0 heat spreading angle always underestimates the peak temperature it seems reasonable to examine whether a smaller heat spreading angle will give better accuracy. Figure 7 shows the effect of using a smaller heat spreading angle of 30^0 for A/B = 0.1. Although the 30^0 heat-spreading approximation significantly reduces the under-estimate of thermal resistance for small l/B, it substantially degrades the accuracy for large l/B ratios where it over-estimates it by up to 45%. Although Figure 7 shows the results for one particular value of the ratio *A/B,* and for a particular reduced value of spreading angle, namely 30^0, the trends shown in Figure 7 are general and apply for all values of A/B and other values of reduced heat-spreading angle.

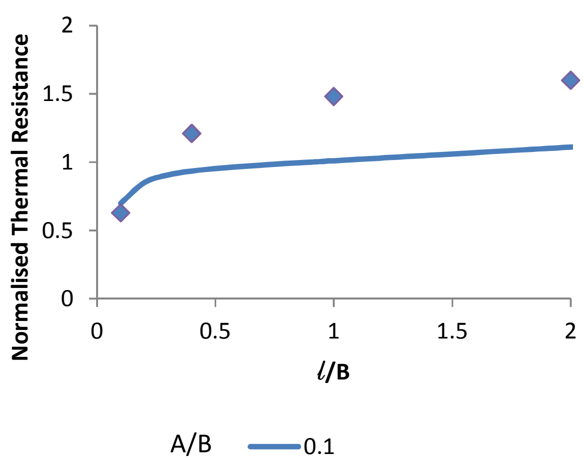

Figure 7. Graph of Normalised Thermal Resistance with 30^0 heat-spreading angle.

APPLICATION OF THE 45⁰ RULE TO MULTIPLE LAYERS

Brook and Smith [3,4] derived an exact analytic solution when the heat sink consisted of multiple layers as shown in Figure 4, but the expression is more complex than that given by Equation 4 and so is not reproduced here, but it is still fairly easy to compute using Excel™. The 45^0 rule permits a very quick and easy way of determining the overall thermal resistance for a multi-layer structure. However, it is essential to account for the change of spreading angle which occurs at the junction of two materials with different thermal conductivities. Figure 8 shows the geometry of the problem.

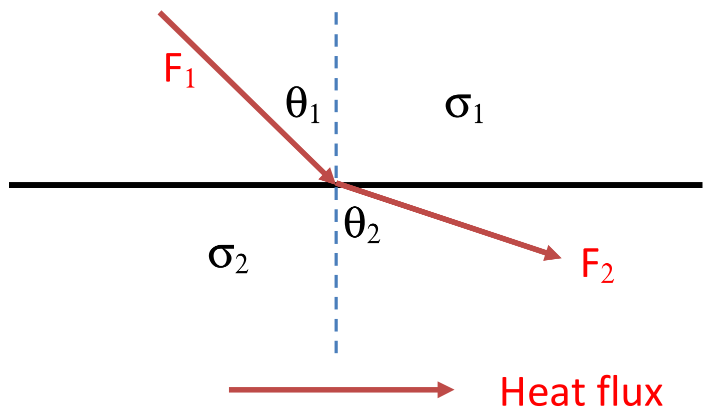

Figure 8. Change of heat spreading angle at the junction of two materials with different thermal conductivities

The relationship between θ_1 and θ_2 is easily derived. There can be no build-up of heat flux at the interface and hence the vertical components must be equal i.e.

$$F_1 \cos\theta_1 = F_2 \cos\theta_2 \qquad (7)$$

Also, the temperature at any horizontal position along the interface must be the same on either side of the junction. Hence if the horizontal component of the heat flux travels a distance Δx then

$$\frac{F_1 \sin\theta_1 \Delta x}{\sigma_1} = \frac{F_2 \sin\theta_2 \Delta x}{\sigma_2} \qquad (8)$$

Solving equations (7) and (8) gives

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\sigma_1}{\sigma_2} \qquad (9)$$

Equation (9) is the thermal analogue of Snell's law in optics. Holway and Adlerstein cautioned about using the 45^0 heat-spreading approximation if the two adjacent materials had very different thermal conductivities. To illustrate the importance of observing this note of caution, consider the two structures shown in Figure 9.

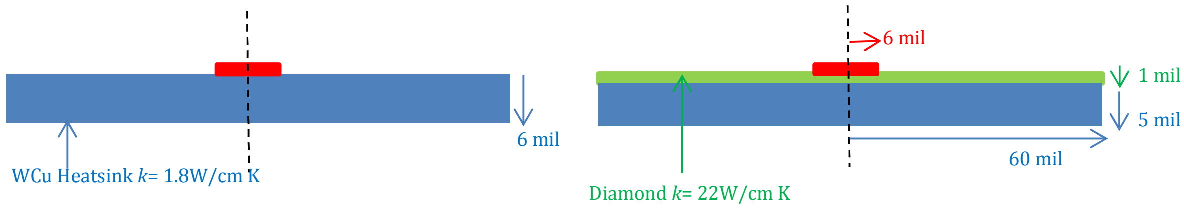

Figure 9. (a) Single-layer structure of WCu, (b) top 1mil of WCu is replaced with diamond, otherwise the structure and dimensions are identical.

It seems obvious that the structure in Figure 9(b) should have a lower thermal resistance than that in Figure 9(a) since the two structures are identical in every respect except for the fact that the top 1 mil of WCu (a common material used as the heatsink in ceramic packages due to its excellent coefficient of expansion match to that of Si) has been replaced with diamond which has a factor of about 10 times higher thermal conductivity than that of WCu. In Figure 9(a) the heat flow is assumed to be 45^0 throughout, in which case the 45^0 heat-spreading rule gives an overall thermal resistance of 6.81^0C/W. In Figure 9(b) the heat flow is assumed to be 45^0 in the diamond and then given by equation (9) in the WCu i.e. $\theta_2 = 4.7^0$, there being almost no heat spreading in the lower region. Application of Equation (5) to the two layers results in an overall thermal resistance of 7.39^0C/W which is higher than for the situation of pure WCu; this is clearly non-sense. The dimensions and materials for the example in Figure 9 have been deliberately chosen to exacerbate the effect.

If $\sigma_1 > \sigma_2$ then $\theta_2 < \theta_1$, in fact if σ_1 is very much larger than σ_2, which is the case with a diamond layer, then θ_2 is close to zero i.e. there is almost no heat spreading in the lower layer and the heat flow is almost pure columnar. This is why the blind application of the 45^0 heat-spreading rule gives an erroneous result. Equation 9 is an exact relationship and so the only way that the 45^0 heat-spreading rule can give a lower value for the thermal resistance of the structure in Figure 9(b) is if θ_1 is significantly greater than 45^0. This is not in contradiction with the statement made immediately beneath equation 4 that the value of σ has no effect on the temperature profile i.e. the heat spreading angle. That statement is true for the situation considered, namely a single layer with a constant temperature at all points on the lower surface, but is not true for multiple layers where the temperature at the interface varies along the interface. This example demonstrates that the heat spreads laterally in the diamond to a much greater extent than 45^0 and shows that even a very thin layer of diamond can significantly reduce the overall thermal resistance. This has been verified practically. Figure 10(a) shows the construction of a conventional 300W VDMOS RF power transistor using a BeO package while Figure 10(b) shows one using a very thin diamond substrate mounted directly on top of the flange [6]. In both cases the number and type of Si die are identical, and both parts deliver a minimum of 300W of RF output power. The conventional part has a specified thermal resistance of 0.4 ^{0}C/W while the diamond substrate version has a thermal resistance of 0.2 ^{0}C/W. If the BeO is replaced with diamond but everything else remains unchanged, then based on the ratio of the thermal conductivity of diamond to WCu then the overall thermal resistance would be close to 0.3 ^{0}C/W while in practice it is 0.2 ^{0}C/W. This verifies the assertion that the heat enters the flange at a much greater spreading angle than 45^0 due to the large difference in the thermal conductivities of diamond and WCu.

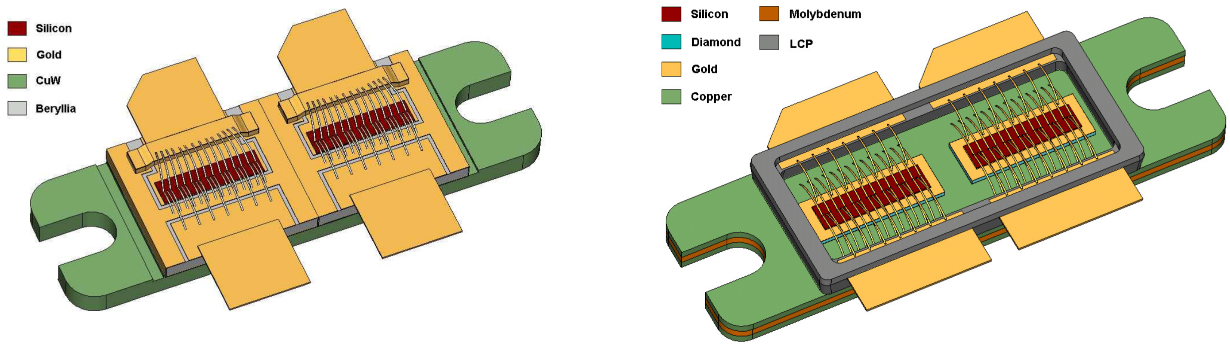

Figure 10. (a) Conventional 300W VDMOS RF power transistor using a BeO substrate, (b) The same transistor using a diamond substrate.

CONCLUSIONS

This paper has shown that the 45^0 heat spreading rule always underestimates the thermal resistance of an RF power transistor, and it can be as much as 40% lower than the real value. The paper has also shown that the 45^0 rule can give misleading results when applied to multi-layer structures if the thermal conductivities differ substantially. Nevertheless, the 45^0 heat-spreading rule remains a useful qualitative tool for the thermal design of individual transistor die as well as complete packaged transistors, but its quantitative estimates of thermal resistance should be used with caution, especially when the heat flow is predominantly by spreading or if layers with widely differing thermal conductivities are used.

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