## Power Handling of Strongly-Coupled Striplines


Abstract

Strong coupling in stripline is readily achieved by the use of partial to complete broadside coupling configuration. The requirement for high even mode impedance leads to narrow lines, when compared with attached transmission lines, and hence to reduced RF current for acceptable temperature rises. The presence of the coupled line, which also carries RF power, adds extra thermal load.

Thermal analysis may be performed, by way of analogy to electrical circuits, to predict maximum power handling in a given configuration. The example of a 2.45GHz equal division coupler demonstrates power limitation due to the coupler is considerably lower than the 50Ω lines that connect to it.

It is shown too that the contribution of dielectric loss is diminished in the coupled line.


### Electrical and Thermal Analogy

The analogy between electrical and thermal analysis can be demonstrated by reference to Fig.1, an analytical representation of a single stripline, in which the red lines denote the stripline conductors, the white space is the dielectric, the blue lines equi-potentials and the black lines flux.

In the synthesis of striplines, calculators determine electrical parameters such as impedance and propagation constant, by considering the blue lines as electrical equi-potentials and the black lines as electrical flux. The same field patterns arise if the white space becomes a conductive medium, in which case the blue lines remain as electrical equi-potentials, but the black lines then become current flux.

The same pattern arises if the structure is considered as a thermal model, where the centre line is held at a different temperature to the ground lines. In this case, the blue lines represent isotherms and the black lines heat flux. We may use this analogy and the existence of stripline calculators as a means of analysing temperature effects in striplines.

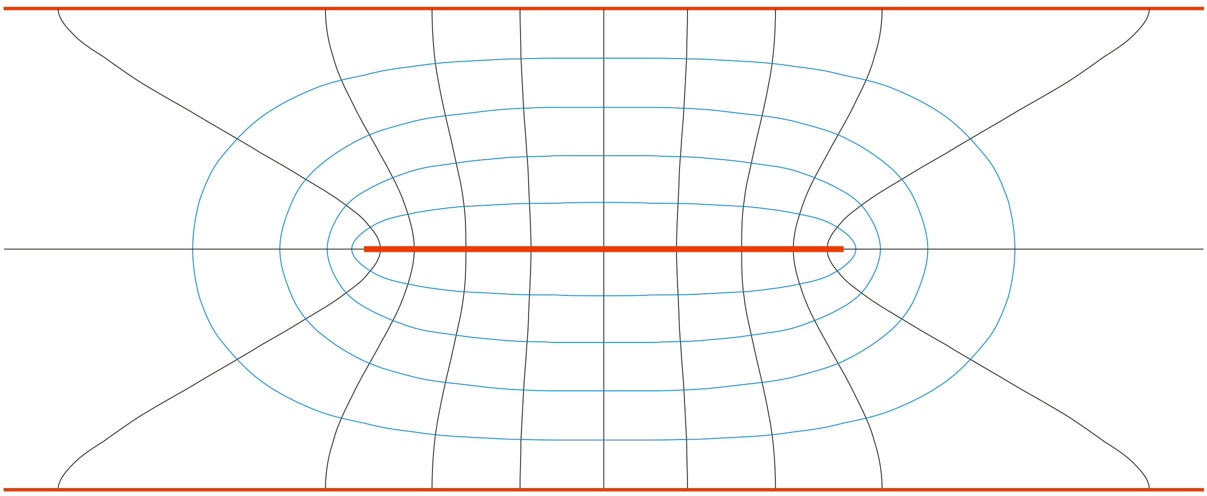

Fig. 1. Field Patterns in Cross-Section of Stripline

In order to demonstrate the analogy, consider first a primitive parallel plate arrangement. From the fundamental definition of capacitance, we have the formula:

$$C = \frac{\varepsilon w l}{d} \qquad (1)$$

where $C$ is the capacitance, $\varepsilon$ is the permittivity, $w$ the width, $l$ the length and $d$ the distance between the plates.

We may also analyse this simple structure in relation to its thermal properties, where one plate is held at a different temperature to the other. The thermal conductance of the structure is given by:

$$K = \frac{\kappa w l}{d} \qquad (2)$$

where $K$ is the thermal conductance, $\kappa$ is the thermal conductivity of the medium, $w$, $l$ and $d$ as before.

Combining (1) and (2) we have:

$$\frac{C}{\varepsilon} = \frac{K}{\kappa} = \frac{wl}{d} \qquad (3)$$

In (3), the quantity *wl/d* is a "shape factor", valid for the parallel plate structure. Other "shape factors" include:

$\frac{2\pi l}{ln\left(\frac{b}{a}\right)}$ Coaxial line, *b*, *a* outer and inner conductor radii respectively

$\frac{\pi l}{cosh^{-1}\left(^{D}/_{2a}\right)}$ Two wire line, *D* centre separation and *a* conductor radius

The scope of analytic solutions in conjunction with useful structures is very limited. Even the parallel plate solution is of only limited use, as it fails to consider fringing fields. A pair of parallel cylinders, one of which may be inside the other, and one of which may be a flat plane in the limiting case, may be analysed analytically as an extension of the coaxial and two-wire line analysis. Approximate techniques have to be used otherwise, including the analysis of striplines.

The analysis has assumed an homogeneous medium in which the fields are present. This assumption is valid for stripline. Inhomogeneous media structures might also be handled, but won't be considered in this paper.

For our purposes, we are interested in solving a 2D problem, as depicted in Fig.1. We are therefore interested in properties per unit length. Choosing the *l* subscript to denote per unit length quantities, (3) may be adapted to:

$$\frac{C_l}{\varepsilon} = \frac{K_l}{\kappa} = \frac{w}{d} \qquad (4)$$

Although the capacitance per unit length of a stripline may be generated by synthesis software, characteristic impedance is its primary output. We have from transmission line theory:

$$C_l = \frac{1}{v_p Z_o} = \frac{\sqrt{\varepsilon_r}}{c Z_0} \text{ F/m} \qquad (5)$$

where $v_p$ is the phase velocity, $Z_o$ the characteristic impedance, $c$ the free space velocity of electromagnetic propagation and $\varepsilon_r$ the relative dielectric constant of the medium. Combining (4) and (5) gives us the expression for thermal conductance per unit length as:

$$K_l = \frac{\kappa\sqrt{\varepsilon_r}}{\varepsilon c Z_o} = \frac{\kappa}{\varepsilon_o\sqrt{\varepsilon_r} c Z_0} \text{ W/°Cm} \qquad (6)$$

Note that this formula is valid for any TEM mode structure, not exclusively striplines.

Thermal Sources

Having determined an expression for the thermal conductance, we now need to determine the loss that gives rise to a temperature increase. There are two sources we are concerned with in connection with striplines. The first is conductor loss (assumed copper) and the other is dielectric loss. Their treatment varies because the first occurs at the boundary and the second within the medium.

Consider first the copper resistance loss. Where it is a small component of the propagation constant, it may be approximated by:

$$Q_C = 2\alpha_c P_i \tag{7}$$

where $Q_C$ is the dissipation due to copper loss, $\alpha_c$ the copper loss component of propagation constant and $P_i$ the incident power.

Copper loss in stripline calculators is usually expressed in dB/m, rather than nepers/m, and one may be calculated from the other by:

$$\alpha_c = \frac{L_c}{20} ln10 \tag{8}$$

where $L_c$ is copper loss in dB/m. Note that copper loss is a strong function of surface roughness, varying by a factor of 2 from very smooth to very rough. It also increases with temperature at a rate of 0.393% per °C, so it is necessary to use a figure relating to the temperature of operation. Note too that copper loss derived from calculators includes a component due to ground plane. For our analysis, we may assume the ground plane is directly cooled and only the temperature rise of the centre conductor matters. Analysis based on calculated total copper loss will therefore lead to a pessimistic power handling figure. As stripline transmission lines are relatively wide compared with coupled lines, the error is greater for the transmission line.

With regard to the dielectric loss, a similar analysis gives rise to:

$$Q_D = 2\alpha_d P_i \tag{9}$$

$$\alpha_d = \frac{L_d}{20} ln10 \tag{10}$$

In the above, $Q_D$ is the dissipation per unit length due to dielectric loss, $\alpha_d$ the dielectric loss component of propagation constant and $L_d$ the dielectric loss per unit length expressed in dB/m. Unlike resistive loss, which includes a complicated function of surface roughness, dielectric loss can be readily evaluated from the dielectric loss tangent. We have from transmission line theory:

$$\alpha_d \approx \frac{G_l Z_0}{2} \tag{11}$$

The parallel conductance parameter can be evaluated from the loss tangent by:

$$G_l = \omega C_l tan\delta \tag{12}$$

As dielectric loss is distributed through the medium, its contribution to centre conductor temperature is lower than it would be had the loss been concentrated in the centre conductor. To understand its effect, consider once again the field patterns in stripline, but adding a couple of loss sources, as shown in Fig.2.

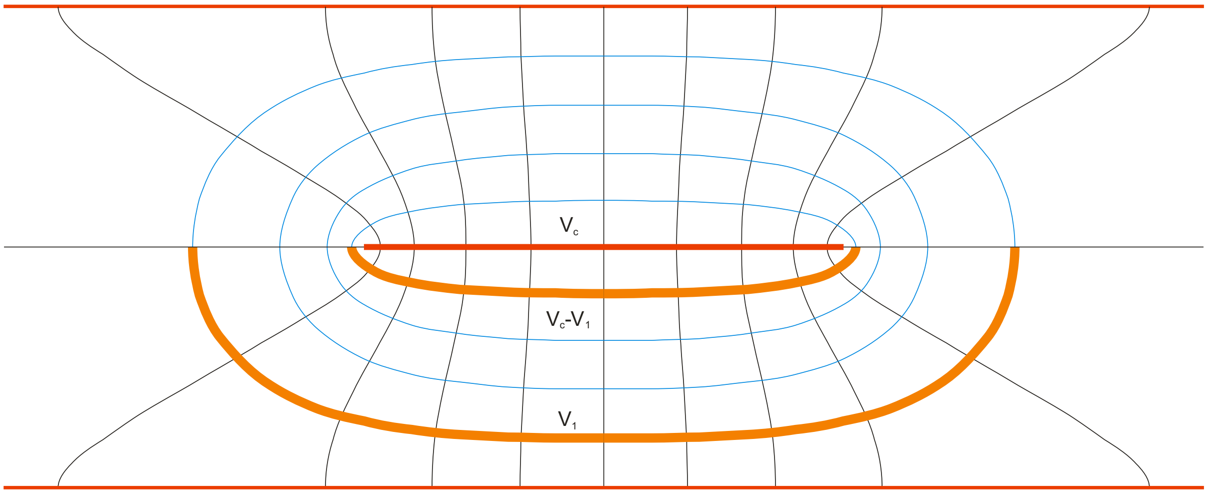


Fig.2 Representation of Dielectric Loss

In addition to the previous potential and flux lines, two bands of loss are shown in orange. These are supposed to occupy an equal elemental voltage. The same flux passes through both bands. As loss is proportional to potential multiplied by flux, the same dissipation occurs in both bands. One band is placed at a mean potential of $V_1$ and the other at $V_c - V_1$. The temperature rise to the $V_c - V_1$ potential is augmented by the additional temperature rise from ground to the $V_1$ potential. As the thermal resistance from the $V_1$ potential to ground is the same as the thermal resistance from the centre conductor to $V_c - V_1$ potential, the aggregate of both loss sources is the same as if a loss equal to one source was placed at the centre conductor.

Now, adding source pairs covering the whole dielectric loss, the result is equal to half the total loss if it was applied to the centre conductor. This is a consequence of the equivalence of the electric and thermal model. An alternative way to express it is to suppose that the dielectric loss is generated at the mid- potential. This is valid for conductor temperature calculations, but not the temperature distribution of the medium.

The two contributions may be combined to determine the centre conductor temperature rise:

$$\Delta T = \frac{Q_C + \frac{1}{2} Q_D}{K_l} \tag{13}$$

Combining (6-10) an expression for power rating can be determined:

$$P_i = \frac{10\kappa\Delta T}{\varepsilon_0\sqrt{\varepsilon_r}cZ_0 \ln 10\left(L_c + \frac{1}{2}L_d\right)}$$

$$= \frac{3273\kappa\Delta T}{\sqrt{\varepsilon_r}Z_0(2L_c + L_d)}\text{ W} \tag{14}$$

<u>Example 1</u>

Determine the power rating of a 50Ω transmission line stripline where ground plane spacing is 6.86mm, at 2.45GHz, where conductor temperature is 100°C above a 40°C case temperature.

The material properties can be taken as $\varepsilon_r$ = 2.2, tan $\delta$ = 0.0007, $\kappa$ = 0.261W/m°C, surface roughness = 3$\mu$m and copper thickness = 35$\mu$m.

Stripline synthesis yields a line width of 5.57mm and analysis then gives $L_c$ = 0.53dB/m and $L_d$ = 0.23dB/m.

Applying the formula of (14) gives a power rating of 893W.

The above example illustrates that in most cases copper loss is greater than dielectric loss. Copper loss can be further reduced by thicker substrate and a wider line, subject to wavelength constraints. Dielectric loss on the other hand doesn't vary with substrate thickness. The effect of dielectric loss is further reduced by its generation away from the centre line.

Junction Cooling

It is anticipated that the narrow lines required in strongly coupled stripline will experience higher temperature rises than the relatively wide connecting transmission lines. A source of cooling in addition to the substrate in the transition region may come from conduction along the copper line. For this analysis, the electrical/thermal analogy may again be employed.

From transmission line theory:

$$Z_0 = \sqrt{\frac{R_l + j\omega L_l}{G_l + j\omega C_l}}$$

Now, when frequency is zero, we have at DC:

$$Z_0 = \sqrt{\frac{R_l}{G_l}} \qquad (15)$$

For the propagation constant:

$$\gamma = \alpha + j\beta = \sqrt{(R_l + j\omega L_l)(G_l + j\omega C_l)}$$

Once again at DC:

$$\alpha = \sqrt{R_l G_l} \qquad (16)$$

Transferring these results to the thermal case. For the junction thermal resistance we get from (15):

$$R_{t50} = \sqrt{\frac{R_{cl}}{K_l}} \qquad (17)$$

In (17), $R_{t50}$ is the thermal resistance at the junction, $R_{cl}$ is the thermal resistance of the copper strip on its own, per unit length and $K_l$ the thermal conductance from (6).

The penetration of cooling effect is the reciprocal of the (16) analogy, so we have:

$$d_p = 1/\sqrt{R_{cl} K_l} \qquad (18)$$

In (18), $d_p$ is the distance the cooling effect has dropped by a factor 1/*e*. It might alternatively by expressed as a "half-life", which equals 0.69 x $d_p$.

Example 2

Determine the cooling effect of the transmission line of Example 1. Use a value of 401W/m°C for copper conductivity.

For the thermal resistance per unit length, use:

$$R_{cl} = \frac{1}{\kappa CSA} = \frac{1}{401 \times 5.57 \times 10^{-3} \times 35 \times 10^{-6}} = 12{,}792°\text{C/Wm}$$

From (6) we get:

*$K_l$=1.326W/m°C*

Applying (17) gives us a thermal resistance of 98.2°C/W and (18) a penetration depth of 7.7mm.

Note that the penetration depth is only a fraction of the length of a quarter wave coupler at the chosen frequency of 2.45GHz, where $\lambda/4$ is 20.6mm.

Coupling Analogy

We now turn our attention to the analysis of couplers. The single capacitance per unit length for a transmission line is replaced with an array of 3 capacitors to represent coupling. Individually, these may then be transformed to the analogous thermal conductances. After this, the effects of dissipation may be applied. This is shown in Fig.3.

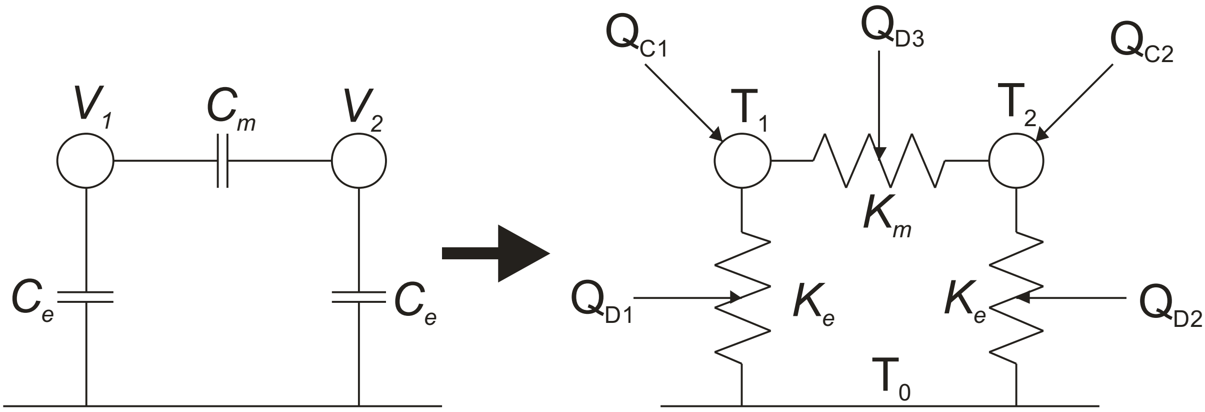


Fig.3 Electrical to Thermal Analogy of Coupler

Coupled transmission line theory gives us for the capacitances per unit length:

$$C_e = \frac{\sqrt{\varepsilon_r}}{cZ_{oe}} \tag{19}$$

$$C_m = \frac{\sqrt{\varepsilon_r}}{2c}\left(\frac{Z_{oe}^2 - Z_0^2}{Z_0^2 Z_{oe}}\right) \tag{20}$$

Comparing these two equations with (4-6) yields:

$$K_e = \frac{\kappa}{\sqrt{\varepsilon_r}\varepsilon_0 c Z_{oe}} \tag{21}$$

$$K_m = \frac{\kappa}{2\sqrt{\varepsilon_r}\varepsilon_0 c}\left(\frac{Z_{oe}^2 - Z_0^2}{Z_0^2 Z_{oe}}\right) \tag{22}$$

The dissipation sources represented by the $Q_{xx}$ quantities in Fig.3 have to be determined from a knowledge of the voltages and currents prevailing at the cross-section where an analysis is made. In a coupler, these vary according to the position, owing to the presence of standing waves. A simplification is available if the position for analysis is at the incident power end. Here, the voltage is determined by the amplitude of RF voltage, and the current similarly. Note that the copper losses apply at the conductors, whereas dielectric losses apply at the mid-point of the thermal conductances.

It is appropriate to take the position for analysis at the incident power point, because stress is greatest here. Incident voltage and current are greatest, as well as coupling line voltage and current.

With regard to the dissipation sources, we have for resistive losses:

$$Q_{C1} = \frac{R_l P_i}{Z_0} \tag{23}$$

$$Q_{C2} = \frac{R_l P_c}{Z_0} \tag{24}$$

Both these equations make use of the calculation of RF current in the adjoining lines. Note that the resistance per unit length $R_l$ refers to the coupled lines.

For the dielectric losses:

$$Q_{D1} = G_e Z_0 P_i \tag{25}$$

In (25), $G_e$ is the loss of the even mode capacitance, given by:

$$G_e = \omega C_e tan\delta \tag{26}$$

Similarly for the other two dielectric loss sources:

$$Q_{D2} = G_e Z_0 P_c \tag{27}$$

$$Q_{D3} = |V_1 - V_2|^2 G_c \tag{28}$$

$$G_c = \omega C_m tan\delta \tag{29}$$

The thermal network in Fig.3 may be treated in the same way as an electrical network, where the thermal conductances perform as electrical conductances, the dissipation sources are current sources and the temperatures are voltages. Further simplification may be afforded by analysing the circuit at the centre frequency of operation, where $V_2$ is in phase with $V_1$ and given by:

$$V_2 = mV_1 \tag{30}$$

*m* is the voltage coupling coefficient, given by:

$$m = \frac{Z_{oe}^2 - Z_0^2}{Z_{oe}^2 + Z_0^2} \tag{31}$$

The voltage coupling coefficient can be further employed to eliminate the coupling elements:

$$\frac{C_c}{C_e} = \frac{G_c}{G_m} = \frac{m}{1-m} \tag{32}$$

With the sources and elements all defined, conductor temperatures may be determined by solving the thermal network in Fig.3. The results are:

$$T_1 - T_0 = [2R_l(1 - m + m^2) + G_e Z_0^2(1 + m - m^2)]P_i/2Z_0K_e \tag{33}$$

$$T_2 - T_0 = [2R_l + (2 - m)G_e Z_0^2]mP_i/2Z_0K_e \tag{34}$$

<u>Example 3</u>

A 3dB overlay coupler centred at 2.45GHz using 6.86mm ground plane spacing and 0.508mm centre substrate thickness, with the same dielectric as in example 1, is predicted to require 2.81mm wide lines with 1.03mm offset [1].

Determine its power rating if the maximum temperature is limited to 140°C with a 40°C ground plane temperature.

Analysis of the 2.81mm wide line indicates on its own it would have an impedance of 74Ω and resistive loss of 0.64dB/m. For a 3dB coupler, $Z_{oe}$ = 120.7Ω and *m* = 0.7071. $G_e$ from (26) and (19) works out as 4.42 x $10^{-4}$ S/m and $K_e$ from (21) is 0.55W/m°C. Rearranging (33) in terms of temperature and applying the values above gives a value of $P_i$ = 295W.

We may also use (34) to evaluate $T_2 - T_0$ = 88°C.

Observe in this last example how much lower the rating for coupled power is, compared with a single line as determined in Example 1. Note too there is not much difference in conductor temperature, even though the coupled line's dominant resistive loss is only half that of the main line.

Junction Cooling – Coupled Case

It would be helpful in view of the high temperature potential of the coupled lines, if the main lines conducted some heat away. A model for this process is shown in Fig.4. Source temperatures $T_i$, $T_c$, $T_1$ and $T_2$ are the temperatures the input line, coupled output line, through line and coupled line attain respectively away from the junction. $T_{t1}$ and $T_{t2}$ are the temperatures at the through and coupled line junctions respectively. $K_{t50}$ is the reciprocal of $R_{t50}$ from (17). It just remains to determine the junction thermal elements $K_{te}$ and $K_{tm}$.

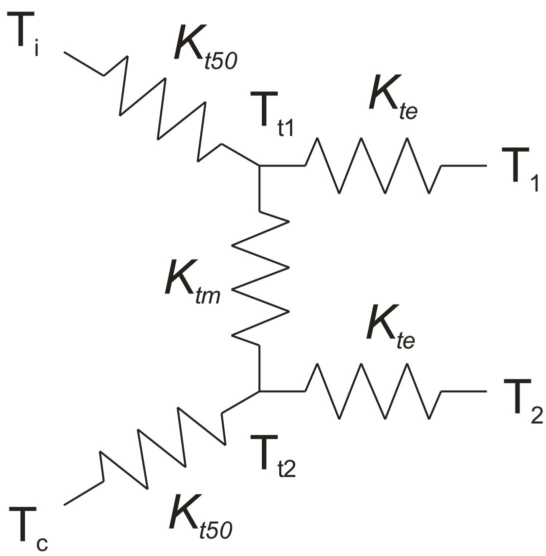


Fig.4 Junction Cooling Model

$K_{te}$ may be readily determined in the same way as $K_{t50}$. From (17) and applying the even mode quantities we have:

$$K_{te} = \sqrt{\frac{K_e}{R_{cl}}} \tag{35}$$

In the case of (35), $R_{cl}$ is the electrical resistance per unit length of the coupled line. In order to determine $K_{tm}$, we need first to determine the odd mode thermal conductance. Examination of Fig.3 allows us to write:

$$K_o = K_e + 2K_m \tag{36}$$

We may show in a similar way to (32) that:

$$K_m = \frac{m}{1-m} K_e \tag{37}$$

Hence:

$$K_o = \frac{1+m}{1-m} K_e \tag{38}$$

The odd mode junction thermal conductance is then:

$$K_{to} = \sqrt{\frac{K_o}{R_{cl}}} = \sqrt{\frac{1+m}{1-m}} K_{te} \tag{39}$$

The mutual junction thermal conductance can then be determined:

$$K_{tm} = \frac{1}{2}(K_{to} - K_{te}) = \frac{1}{2}\left(\sqrt{\frac{1+m}{1-m}} - 1\right) K_{te} \tag{40}$$

It is convenient to write:

$$M = \sqrt{\frac{1+m}{1-m}} = \frac{Z_{oe}}{Z_o} \tag{41}$$

Hence:

$$K_{tm} = \frac{1}{2}(M-1)K_{te} \tag{42}$$

The network of Fig.4 may now be solved, substituting for $K_{tm}$ using (42), giving:

$$T_{t1} = \frac{(T_i K_{t50} + T_1 K_{te})\left[K_{t50} + \frac{1}{2}(M+1)K_{te}\right] + \frac{1}{2}(T_c K_{t50} + T_2 K_{t50})(M-1)K_{te}}{(K_{t50} + M K_{te})(K_{t50} + K_{te})} \tag{43}$$

$$T_{t2} = \frac{(T_c K_{t50} + T_2 K_{te})\left[K_{t50} + \frac{1}{2}(M+1)K_{te}\right] + \frac{1}{2}(T_i K_{t50} + T_1 K_{t50})(M-1)K_{te}}{(K_{t50} + M K_{te})(K_{t50} + K_{te})} \tag{44}$$

Even and odd mode penetration depths may also be inferred. For the even mode:

$$d_{pe} = 1/\sqrt{R_{cl} K_e} \tag{45}$$

For the odd mode:

$$d_{po} = 1/\sqrt{R_{cl} K_o} = \frac{d_{pe}}{M} \tag{46}$$

Odd mode penetration depth is smaller than even mode, indicating the two coupling strips rapidly converge to their ultimate temperature difference, compared to the approach to their final temperature.

<u>Example 4</u>

Determine junction temperatures for the coupler analysed in Example 3, assuming the same input power of 295W, and the connecting transmission lines are the same as Example 2. Normalise temperatures to $T_0 = 0$.

From Example 3, we have $T_1$ = 100°C, $T_2$ = 88°C. With the lower incident power, $T_i$ reduces in proportion to the value determined in Example 2, and is 33°C. As the coupler is an equal power splitter, the coupled transmission line temperature becomes half of $T_i$ or 16.5°C.

Taking the reciprocal of (17) we get $K_{t50}$ = 0.01018W/°C.

In order to determine $K_{te}$, the thermal resistance of the coupled line needs to be evaluated. Proceeding similarly to Example 2, we have:

$$R_{cl} = \frac{1}{\kappa CSA} = \frac{1}{401 \times 2.81 \times 10^{-3} \times 35 \times 10^{-6}} = 25{,}526\text{°C/Wm}$$

Applying (35), with $K_e$ already determined, we get $K_{te} = 4.653 \times 10^{-3}$ W/°C. For an equal division coupler, $M = \sqrt{2}+1$. Applying (43) and (44) in turn we get:

$$T_{t1} = 51.7°C \qquad T_{t2} = 41.3°C$$

For penetration depths, using (45) and (46) in turn we get:

$$d_{pe} = 8.5\text{mm} \qquad d_{po} = 3.5\text{mm}$$

The substantially cooler transmission lines are effective in reducing junction temperatures, but the relief is limited. Temperature will increase away from the junction before the reduced RF amplitude reduces stress.

References

[1] Shelton J., "Impedances of Offset Parallel-Coupled Strip Transmission Lines", *IEEE Trans. Microwave Theory and Techniques*, Vol. MTT-14, No. 1, Jan. 1966, pp. 7-15.