Using air lines as references for VNA phase measurements

Stephen Protheroe and Nick Ridler
Electromagnetics Team, National Physical Laboratory, UK
Email: Stephen.protheroe@npl.co.uk

Abstract
Air lines are often used as impedance references to evaluate the magnitude uncertainty in vector network analyser (VNA) calibrations using the so-called ripple technique. However, there is as yet no comparable technique for assessing the associated phase uncertainty. By means of practical measurement, this paper demonstrates the difficulty in assessing phase uncertainty using air lines and describes why this is caused by a lack of knowledge concerning the conductor loss in the lines.

1 Introduction

Precision air dielectric coaxial transmission lines (or air lines, for short) form a key component in current recommended techniques for assessing magnitude uncertainty in vector network analyser (VNA) calibrations [1], but no similar technique currently exists for assessing the phase uncertainty. Since air lines are already used to assess magnitude uncertainty, it would be convenient if they could also be used to assess phase uncertainty.

If the mechanical length of an air line is known accurately, then it is straightforward to calculate the expected phase shift due to its insertion in a circuit. Consider a length, l, of air line connected between the port 1 and port 2 reference planes of a VNA:

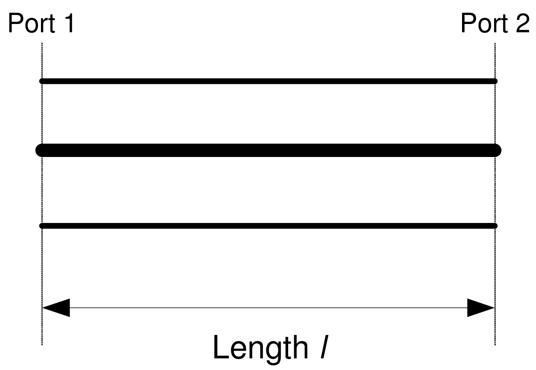

Figure 1: schematic diagram of an air connected between the reference planes of a VNA.

The electrical delay is the time taken for the signal to travel between the ports:

$$Delay(s) = \frac{l\sqrt{\varepsilon_r}}{c} \qquad (1)$$

where

ε_r = relative permittivity of line's dielectric, ε_r = 1.000649 for air at 23 ºC
c = speed of light ≈ 2.997925 × 10^8 m.s^{-1}.

For a coaxial line, the phase response, ϕ, is linear with frequency:

$$\phi(rad) = 2\pi f \times delay = \omega \times delay \tag{2}$$

where

ω = angular frequency (rad.s^{-1})
f = frequency (Hz).

Hence

$$\phi = \frac{2\pi f l \sqrt{\varepsilon_r}}{c} = \beta_0 l \tag{3}$$

where

β_0 = phase constant (rad.m^{-1}) $= \frac{2\pi f \sqrt{\varepsilon_r}}{c}$.

The measured phase ($-\pi < \theta < \pi$) can be found from the phase response as[1]:

$$\theta(rad) = \pi - \text{mod}(\phi + \pi, 2\pi)\ . \tag{4}$$

Conversely, the phase response may be calculated from the measured phase:

$$\phi(rad) = \text{mod}(2\pi - \theta, 2\pi) + 2\pi n \tag{5}$$

where n is an integer (n = 0, 1, 2, …).

The electrical length of the line may therefore be calculated from the measured phase using:

$$l = \frac{c(2\pi n - \theta)}{2\pi f \sqrt{\varepsilon_r}} = \frac{(2\pi n - \theta)}{\beta_0}\ . \tag{6}$$

Since there are multiple solutions to equations (5) and (6), the correct value for n can be determined by finding a length that is within a given tolerance of the nominal length. This

[1] The modulo arithmetic operator mod($\phi + \pi$, 2π) returns the remainder after ($\phi + \pi$) is divided by 2π.

tolerance is dependent on the frequency (wavelength) of the measurement, e.g. at 18 GHz it will be approximately 20 mm.

2 Measurements

Three 50 ohm air lines fitted with precision 7 mm connectors were selected for this investigation: two nominal 100 mm lines from different manufacturers (i.e. Hewlett Packard and Maury Microwave) and one nominal 300 mm line (manufactured by Maury Microwave). The mechanical lengths of these lines were measured by a UKAS-accredited laboratory.[2]

The *S*-parameters of the lines were measured using NPL's primary national standard VNA measurement system to obtain the best possible accuracy. The electrical length of the lines was calculated using equation (6) from both S_{21} and S_{12} phase data and compared with the mechanical length. Figure 2 shows a plot of the difference in length between the electrical and mechanical determinations, as a function of frequency.

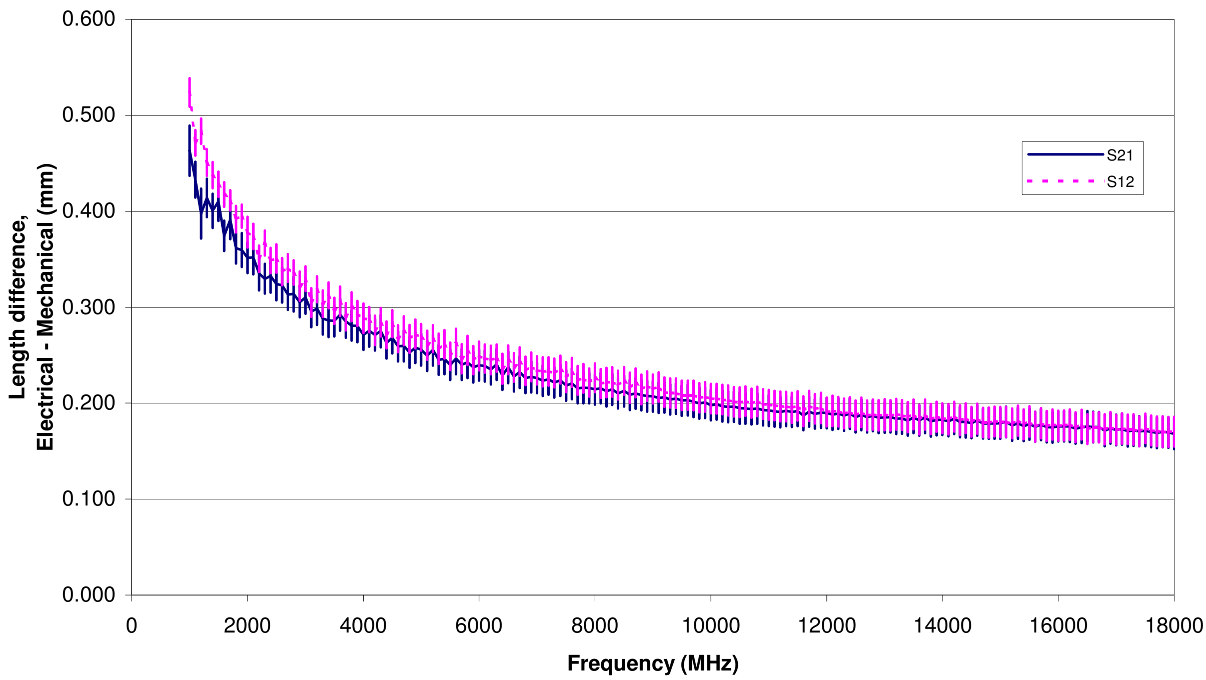

Figure 2: showing the difference in length between the electrical and mechanical determinations of the length for the 300 mm line.

[2] A correction was applied to these length measurements to take account of the difference in temperature between the UKAS-accredited calibration laboratory temperature (i.e. 20 °C) and NPL's VNA laboratory temperature (23 °C).

This Figure shows that the electrical determination of the line's length produces a longer length value than the mechanical determination at all frequencies. The length difference is also frequency dependent. Finally, Figure 2 also shows that the data derived from S_{21} is essentially the same as that derived from S_{12} (as is to be expected for a reciprocal device) and so only S_{21} data will be shown in the plots that follow.

Figure 3 shows a similar plot to Figure 2 except that the data for the two 100 mm lines has been included. As before, there is a systematic difference between the electrical and mechanical determinations of the length – the electrical determinations measured the lines to be longer than their mechanical length. However, this difference is less for the 100 mm lines than it is for the 300 mm line.

Again, as before, the length difference is frequency dependent. However the frequency dependence is less for the 100 mm lines than it is for the 300 mm line.

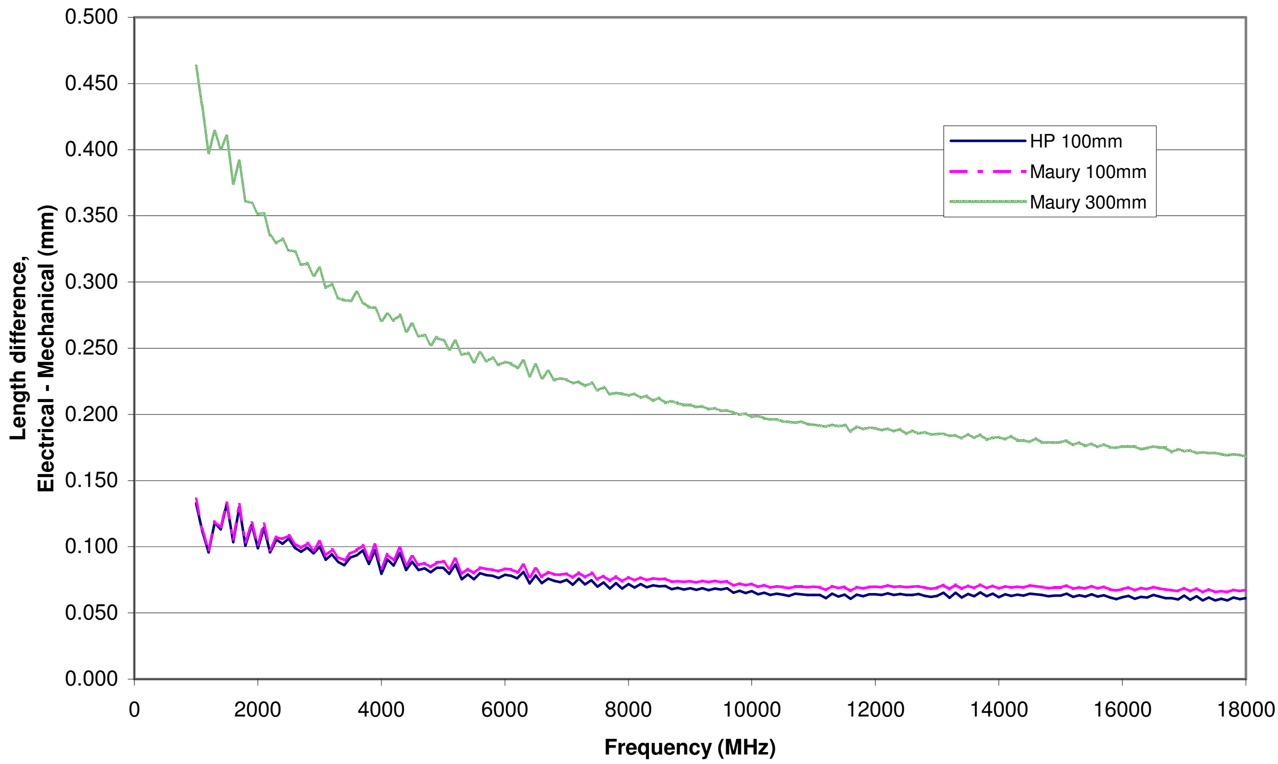

Figure 3: showing the difference in length between the electrical and mechanical determinations of length for the two 100 mm lines and the 300 mm line.

3. Lossless versus lossy lines

The theory presented so far in this paper has assumed that the air lines are lossless. However, in practice, this will not be the case. To illustrate this, Figure 4 shows measurements of the linear magnitude of S_{21} (i.e. equivalent to the attenuation, or 'loss') of the lines as a function of frequency.

A consequence of this loss is to increase the value of the phase constant from its lossless value, β_0, by an amount that is dependent on frequency. For example, although $\beta_0 = 1201.22° \text{ m}^{-1}$ at 1 GHz, the electrical length measurement indicates a value that is somewhat higher.

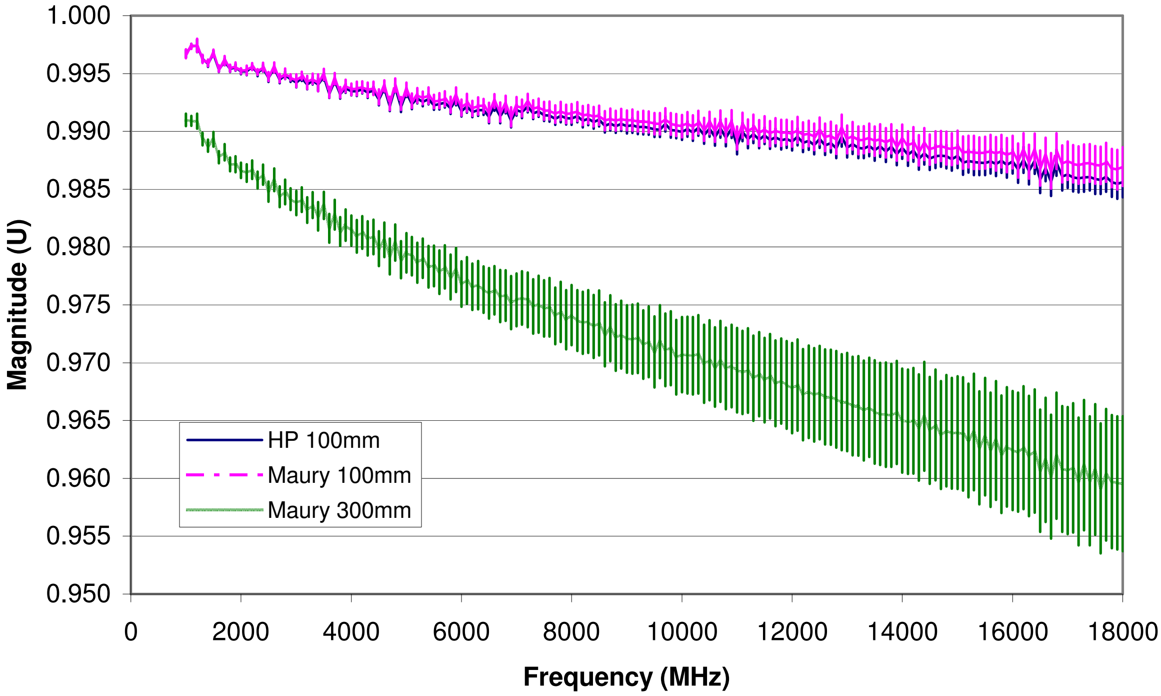

Figure 4: S_{21} linear magnitude, with uncertainties shown as 'error bars', for each air line.

From the measured phase data and mechanical length, equation (6) can be used to determine a value for the actual phase constant, β, taking into account the loss in the line:

$$\beta = \frac{(2\pi n - \theta)}{l} \tag{7}$$

If we now consider a lossy coaxial line in detail, then:

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \tag{8}$$

where

γ = propagation constant

α = attenuation constant (Np.m^{-1})

R = series resistance ($\Omega.\text{m}^{-1}$)

L = series inductance (H.m^{-1})

G = shunt conductance (S.m^{-1})
C = shunt capacitance (F.m^{-1})

It has been shown that [2]:

$$R = 2\omega L_0 d_0 \left(1 - \frac{k^2 a^2 F_0}{2}\right)$$
$$L = L_0 \left[1 + 2d_0 \left(1 - \frac{k^2 a^2 F_0}{2}\right)\right]$$
$$G = \omega C_0 d_0 k^2 a^2 F_0$$
$$C = C_0 \left(1 + d_0 k^2 a^2 F_0\right)$$

where

L_0 = series inductance of lossless line (H.m^{-1}) $= \dfrac{\mu \ln\left(b/a\right)}{2\pi}$

C_0 = shunt capacitance of lossless line (F.m^{-1}) $= \dfrac{2\pi\varepsilon}{\ln\left(b/a\right)}$

a = radius of line's centre conductor (m)
b = radius of line's outer conductor (m)
k = angular wavenumber, $k = 2\pi/\lambda$ (rad.m^{-1})
ε = permittivity, $\varepsilon = \varepsilon_0\varepsilon_r$ (F.m^{-1})
ε_0 = permittivity of free space, $\varepsilon_0 = c^2\mu_0$ (F.m^{-1})
μ = permeability, $\mu = \mu_0\mu_r$ (H.m^{-1})
μ_0 = permeability of free space, $\mu_0 = 4\pi \times 10^{-7}$ (H.m^{-1})
μ_r = relative permeability of line's dielectric, $\mu_r = 1$ for air
F_0 and d_0 are constants defined as:

$$F_0 = \frac{\left(b^2/a^2\right) - 1}{2\ln\left(b/a\right)} - \frac{\left(b/a\right)\ln\left(b/a\right)}{\left(b/a\right) + 1} - \frac{1}{2}\left[\frac{b}{a} + 1\right]$$
$$d_0 = \frac{\delta_s\left(1 + \left(b/a\right)\right)}{4b\ln\left(b/a\right)}$$

where

δ_s = skin depth (m) $= \sqrt{\dfrac{\rho}{\pi f \mu}}$

and

ρ = resistivity (Ω.m).

Since the radii *a* and *b* are known for these lines, equation (7) and the imaginary part of equation (8) can be equated and a value for the resistivity can be determined. Figure 5 shows values of resistivity determined in this way for the three air lines under investigation.

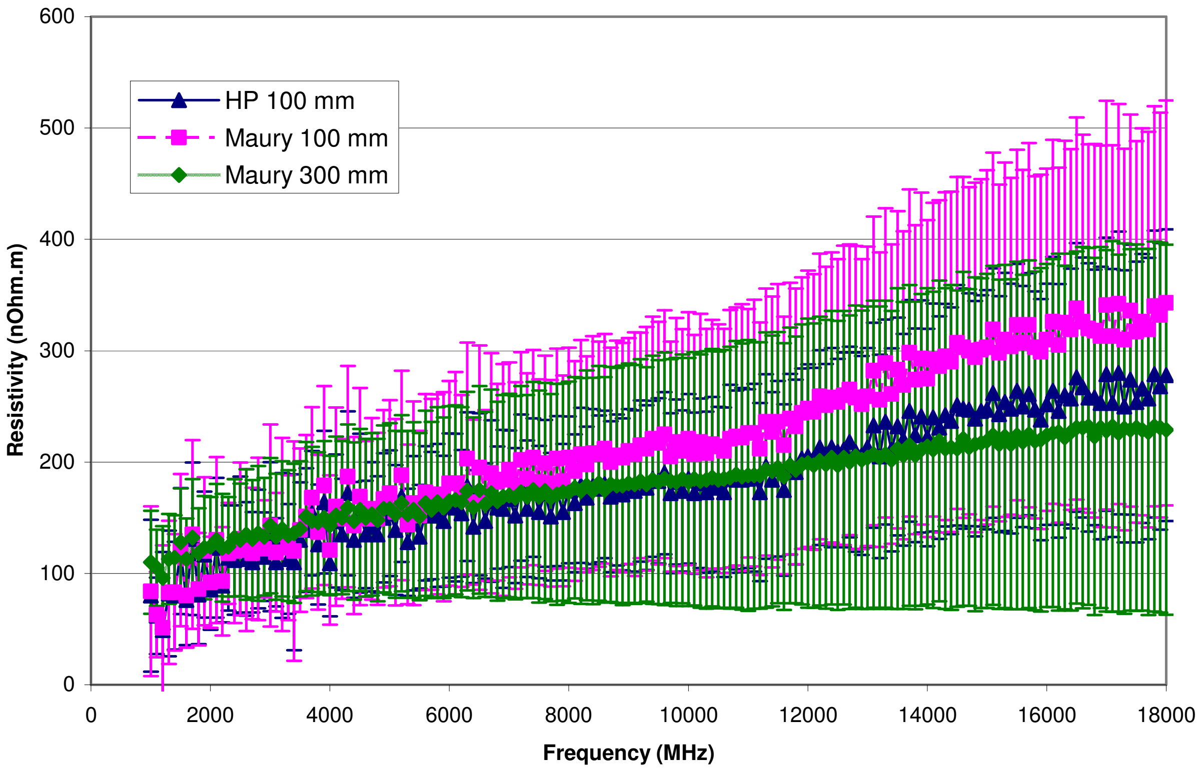

Figure 5: values of resistivity for each air line as a function of frequency.

The uncertainties shown in Figure 5 have been calculated by observing the changes in the calculated resistivity due to offsetting each of the calculation's input parameters, in turn, by its standard uncertainty. These 'component uncertainties' are then combined using a root-sum-of-squares approach.[3] The error bars on the graph show the uncertainties at a 95% level of confidence (i.e. using a coverage factor, $k = 2$).

4. Discussion

Figure 5 shows the values of resistivity for each air line that are needed to cause the electrical and mechanical determinations of each line's length to become equal (to within the stated uncertainties). It is interesting to note that the values of resistivity calculated for each air line agree with each other to within the stated uncertainties. The calculated resistivity also appears to rise slightly with frequency whereas perhaps one might expect this to remain effectively

[3] A sensitivity analysis of the model derived from equations (6) and (7) has shown that the dominant component to the overall uncertainty is due to the measured phase. Other components, such as the measured mechanical length and temperature effects, were found to have a negligible effect on the overall uncertainty of the resistivity determination.

constant over this range of frequencies. However, any such frequency dependence is only slight and well within the stated uncertainties. In fact, the results could be further summarised by choosing a value of approximately 150 nΩ.m, which could be applicable across this entire frequency range.

The above experimentally determined resistivity value of 150 nΩ.m can be compared with 'textbook' values for suitable materials. For example, in [3], a value for the resistivity of brass at room temperature is given as 63 nΩ.m. This is considerably less than the value determined in this paper. This could be for the following reasons:

1) It is not certain that the air lines' conductors are made of brass, although brass is a material that can be used to fabricate such air lines [4]. Other suitable materials (e.g. beryllium copper, which is also often used to fabricate air lines) will have different values of resistivity;
2) The textbook value of 63 nΩ.m has probably been derived from measurements made at DC and therefore may not be representative of the resistivity at high frequencies. However, the lack of significant frequency dependence observed in this paper tends to suggest that values derived at DC will not be too different from values at higher frequencies;
3) Textbook values of resistivity usually refer to bulk samples of materials, whereas the metal used for air lines is often formed by depositing layers using electroplating and machining techniques. The degree of compactness and surface finish, along with a tendency for electroplated surfaces to become porous, will cause the resistivity to vary significantly from a textbook value of a bulk sample. This will usually result in an increase in the loss (and hence the resistivity) of the material used to construct the line [5]. Such an increase is consistent with the discrepancy observed above.

For the above reasons, it is desirable to establish a convenient method for determining the resistivity of a particular line under the conditions to which it is used. The development of such a method is currently work-in-progress at NPL and is based on an earlier experimental technique [6]. Early indications suggest that the value of 150 nΩ.m is not unrealistic for the lines considered in this investigation [7].

5. Conclusion

The practical difficulty of using air lines as references for VNA phase measurements has been demonstrated. The electrical length will appear longer than the mechanical length of the line if the assumption that the line being used is lossless. Air lines are therefore only useful as accurate phase references if their loss is considered. This requires an accurate knowledge of the resistivity of the conductors of the line and this is likely to be a property of each individual line in question.

When a method becomes available to reliably and conveniently determine the loss in a given line, then this paper has shown that these lines will become suitable reference devices for verifying VNA phase measurements. This will provide a significant improvement to the current international guidelines for evaluating VNA measurements given in [1].

6. References

[1] "EA Guidelines on the Evaluation of Vector Network Analysers (VNA)", *European co-operation for Accreditation*, publication reference EA-10/12, May 2000. (Available from: www.european-accreditation.org.)

[2] W C Daywitt, "First-order symmetric modes for a slightly lossy coaxial transmission line", *IEEE Trans*, **MTT-38**(11):1644-1651, Nov 1990.

[3] Kaye & Laby: http://www.kayelaby.npl.co.uk/general_physics/2_6/2_6_1.html

[4] "Type 900 air lines", *The General Radio Experimenter*, **37**(11):7-8, Nov 1963.

[5] B O Weinschel, "Errors in coaxial air line standards due to skin effect", *Microwave Journal*, **33**(11):131-143, Nov 1990.

[6] G J Kilby and N M Ridler, "Comparison of theoretical and measured values for attenuation of precision coaxial lines", *IEE Electronics Letters*, **28**(21):1992-1994, Oct 1992.

[7] C P Eiø and N M Ridler, "Characterising air lines using VNA loss measurements", *BEMC 2005 conference digest*, National Physical Laboratory, November 2005.