Solution looking for a problem: The Fe^{3+}:sapphire whispering-gallery-mode 12.04 GHz maser oscillator, an X-band microwave source of extremely low phase noise.

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According to Leeson's model, a loop oscillator comprising a high-Q resonator and a low-noise sustaining amplifier will exhibit low phase noise. The cryogenic Fe^{3+}:sapphire Whispering-Gallery-mode 12.04 GHz Maser Oscillator, or "WhiGMO" for short, provides these two ingredients in the extreme: a resonator with a Q of around one billion in conjunction with extremely quiet solid-state maser amplification. In contrast to (tunable) ruby masers, no external dc magnetic field need be applied; the WhiGMO is a "zero-field" maser. Its speculated and measured performance, its operational requirements and limitations, and a few of it more obvious applications (as a frequency/phase reference) shall be discussed. Some less-obvious potential applications shall then be solicited from the audience!

The performance of an oscillator comprising a resonator in loop with a restoring amplifier is governed by Leeson's formula for the oscillator's phase noise as a function of the carrier-offset frequency f:

$$S_\varphi(f) = [1 + \frac{1}{f^2}(\frac{v_0}{2Q})^2]S_\psi(f); \tag{1}$$

where here v_0 is the oscillator's absolute operating (i.e. the carrier) frequency, Q is the quality factor of the resonator, and $S_\psi(f)$ is the restoring amplifier's phase noise. This formula indicates that, to realize oscillators with the lowest phase-noise, one needs high-Q resonators and low-phase-noise restoring amplifiers. Not captured by Leeson's formula, is that, for offset frequencies below 10 Hz, corresponding to time intervals greater than 0.1 s, the oscillator's frequency stability is often limited by "technical" noise sources: fluctuations/drift in temperature, the ageing of materials, variations in the magnetic field, mechanical vibrations/tilting ... that all cause the center frequency of the resonator's operational mode to fluctuate. Fractional frequency stabilities better than 1×10^{-14} over temporal intervals <100 s can be achieved at microwave frequencies with oscillators incorporating cryogenic (<10 K) electromagnetic resonators exhibiting Q values in excess of 100 million[1]. Such oscillators have been used successfully as 'flywheels' for cold-atom frequency standards [2], as reference oscillators for (close-in) phase-noise measurements[3], and in tests of fundamental physics (e.g. Lorentz invariance[4]).

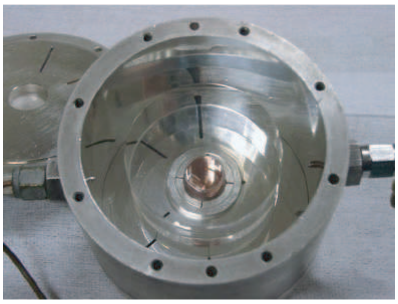

FIG. 1: *View of a typical cryogenic sapphire resonator with the lid of its (silver-plated) microwave can removed.)*

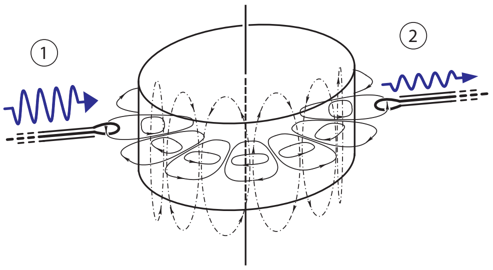

FIG. 2: *The whispering-gallery mode of a cryogenic sapphire resonator, with its input and out couplers to realize a two-port ultra-narrow-band loop filter.*

In recent years, the most actively studied resonators have been those based on whispering-gallery (WG) modes, supported (electromagnetically speaking) on sapphire cylinders or rings maintained at a temperature above 4.2 K[5–7]; figure 1 shows such a device. Here, the bulk of the WG mode's field energy resides just within the curved outer cylindrical wall of the sapphire monocrystal –see the right-hand side of figure 3. A Pound-stabilized-loop oscillator (PSLO)[8] is built around the WG-mode resonator, with the oscillator's sustaining amplifier and phase modulator(s) located outside of the cryostat. A PSLO is thus a spatially extended system; two microwave lines, each typically > 1 m in length, join the cryogenic resonator and room-temperature electronics together in a loop. Moreover, to achieve stabilities at the 1×10^{-14} level, additional circuits supporting the control of the resonator's temperature and received microwave power[9] are required.

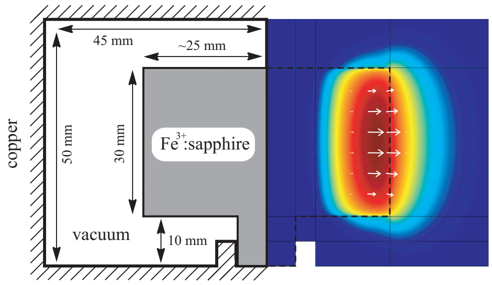

FIG. 3: *The mechanical geometry and electromagnetic morphology of a particular cryogenic sapphire resonator –which can function as a WhiGMO*

The electromagnetic resonator in a PSLO arrangement, as depicted in figure 2; functions as a purely passive, linear single-pole band-pass filter (except potentially for a slight power-dependent frequency shift). In contrast, we describe here an above-threshold maser[10] oscillator, i.e. an *active* resonator, whose requisite amplification is achieved through the interaction between a whispering-gallery mode and a collection of ($\sim 10^{15}$) paramagnetic ions that exhibit an electron spin resonance (ESR). These ions are located, in space, within the WG mode's field profile (fig. 3), and the WG mode is located, in frequency, within the ESR's lineshape. Compared to Pound-stabilized loop oscillators, our incorporation of maser gain within the oscillator's frequency-determining element represents a fundamentally different approach; figure 4 attempts to capture, emblematically, this essential difference.

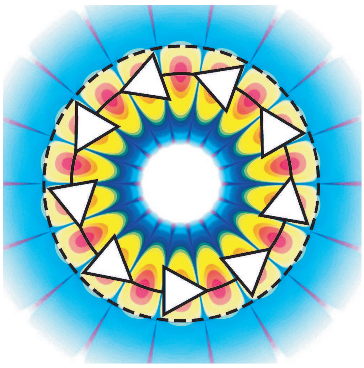

FIG. 4: *The WhiGMO idea: a whispering-gallery mode with intrinsic, distributed microwave gain: WG mode, in effect, constitutes the loop of the WhiGMO's equivalent loop oscillator.*

Our whispering-gallery(-mode) maser oscillator, henceforth 'W[hi]GMO', may be regarded as a free-running loop oscillator, whose loop is the (closed) path taken by its (signal) WG mode through space, and whose amplifier is continuously distributed around this loop/mode. Some immediately apparent advantages are: (i) the rigidity and compactness of the all-sapphire oscillator loop enables its electromagnetic length (hence the WhiGMO's frequency –as determined by the Barkhausen condition) to be kept extremely constant; (ii) unlike a ruby maser, no d.c. magnetic bias field need be applied –the WhiGMO is a "zero-field" maser; (iii) compared to a PSLO, the WhiGMO comprises fewer essential components, *viz.* just the sapphire cylinder and its associated electromagnetic pump- and signal-mode couplers; there is no Pound frequency servo; there are no cables or coupling structures between a spatially separated amplifier and resonator.

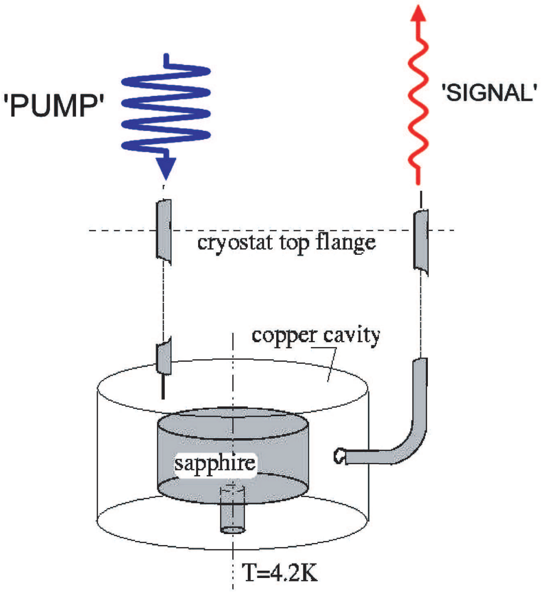

FIG. 5: *Functional principle of the Fe^{3+} WG-mode maser oscillator: 31.3 GHz pump in, 12.04 GHz signal out*

Half a century or so ago, the study of electron spin resonance in solids led to the development of solid-state masers [10–12] as extremely low-noise amplifiers for applications in satellite communications and radio astronomy. Sapphire crystals deliberately doped with Fe^{3+} ions, as opposed to Cr^{3+}, were studied by a few groups[13, 14]; even a few maser amplifiers based on Fe^{3+}:sapphire were demonstrated[15, 16].

Our electromagnetic resonator (see fig. 5) contains a monocrystal of HEMEX-grade[17] sapphire that comprises a main cylinder, 50 mm in diameter and 30 mm high, with a smaller, coaxially adjoining cylinder (its 'spindle') for support; the monocrystal is mounted coaxially within a cylindrical copper cavity, whose interior walls are silver-plated. This monocrystal supports various whispering-gallery modes; but only two of them, both quasi-transverse-magnetic (WGH) in character, are presently relevant: (i) a fundamental (i.e. with no axial or radial nodes) 17th-azimuthal-order WGH mode, at approx. 12.038 GHz and (ii) a different, as yet unidentified WGH mode of considerably higher azimuthal-order at approx. 31.339 GHz. [Both of these frequencies refer to near-4.2 K operation.] These two modes shall henceforth be referred to as the 'signal' and 'pump' modes, respectively. The former is excited by an appropriately positioned and oriented loop probe (sensitive to the magnetic field's azimuthal component), the latter by a stub antenna (sensitive to the electric field's axial component). The surrounding cavity is mounted within a vacuum can on the end of a cryogenic insert, which is loaded into a large liquid-helium dewar. Microwave transmission lines, each comprising several lengths of semi-rigid RG-405 coaxial cable joined by SMA connectors and feedthroughs, connect each of the active resonator's two probes to terminals on the insert's top plate.

Though our sapphire monocrystal was not intentionally doped, ferric iron ions (Fe^{3+}) lie within it as residual impurities, substituting for Al. Estimates in the literature for the concentration of iron (presumably as Fe^{3+}) in samples of an unspecified grade of nominally undoped HEM-grown sapphire [18], and in dielectric resonators made from the HEMEX grade of the same [19], differ by orders of magnitude; we believe the concentration of Fe^{3+} in our piece of HEMEX to be a few parts per million.

The Fe^{3+} ion's three paramagnetic energy levels at zero d.c. magnetic field are represented in fig. 6(a). Each of these is in fact a degenerate Kramers doublet [15, 20]; the $S_z = \pm 5/2$ and $\mp 1/2$ spin states are also mixed slightly together. Transitions between these three levels are (thus) all allowed, though the level-crossing pump transition is rather weak[14]; their linewidths are all expected to be a few or several tens of MHz.

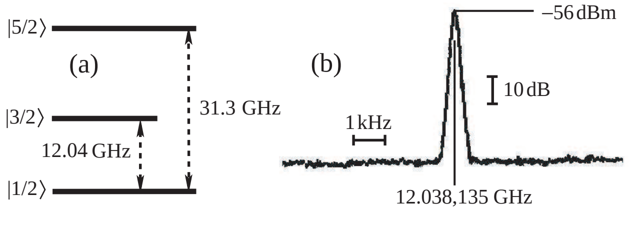

FIG. 6: (a) *Energy-level diagram for* Fe^{3+} *in sapphire at zero applied magnetic field and* (b) *the unamplified maser signal output as observed on a spectrum analyzer (resolution bandwidth* = 100 Hz*).*

Our WhiGMO exploits Bloembergen's classic three-level scheme[11], but at zero applied magnetic field; $|1/2\rangle \leftrightarrow |3/2\rangle$ transitions are stimulated by the resonator's signal mode, whose frequency lies near the center of the signal transitions' lineshape; similarly, $|1/2\rangle \leftrightarrow |5/2\rangle$ transitions are stimulated by the resonator's pump mode. The latter was driven (via its corresponding coupling probe and transmission line) by the output from an Agilent E8254A microwave frequency synthesizer. When set to a frequency of 31.339 GHz, at an output power level of 2 dBm, a -56 dBm maser signal at approx 12.038135 GHz could be detected at the insert's top plate –see fig. 6(b).

The maser oscillator's output signal was amplified by 70 dB then mixed (with a doubly balanced mixer) against the signal from a second microwave synthesizer referenced to a hydrogen maser. The resulting beat-note was sent to a high-resolution frequency counter (HP 53132A). By slowly increasing the resonator's temperature whilst monitoring this counter, we observed the WhiGMO's signal frequency to turn over (a maximum) at a temperature of approx. 7.939 K. The resonator's temperature was then stabilized at this turn-over and the beat-note frequency measured against time. The corresponding fractional-frequency Allan deviation[21] was subsequently computed, with the result shown in fig. 7.

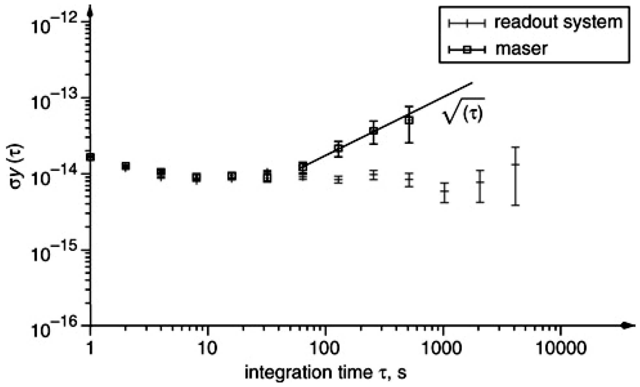

FIG. 7: *Frequency stability of W(hi)GMO; note that frequency stability of the measurement system (reference synthesizer) dominates stability for time intervals below 100 s.*

The stability of the reference synthesizer limited the resolution of our measurement for sampling intervals $\tau <$ 80 s. Nevertheless, a minimum in the Allan deviation below 10^{-14} at around 30 s could be obtained. Assuming a spin-lattice relaxation time ('T_1') of a few ms[14], the measured -56 dBm level of maser signal power is consistent with a concentration of Fe^{3+} ions at the the parts-per-million level provided spectral hole burning (due to inhomogeneous broadening) is taken into account. the $|1/2\rangle \leftrightarrow |5/2\rangle$ transition was not fully saturated at the 2 dBm level of applied pump power used.

Since two WhiGMOs have yet to be operated simultaneously and be beaten together, the WhiGMOs true phase noise has yet to be quantified. As a reference point, the phase noise of two conventional (Pound-locked loops, using conventional room temperature microwave restoring amplifiers) cryogenic sapphire oscillator operating at near 9.2 GHz [22] is shown in figure 8 [see ref. [22] for details].

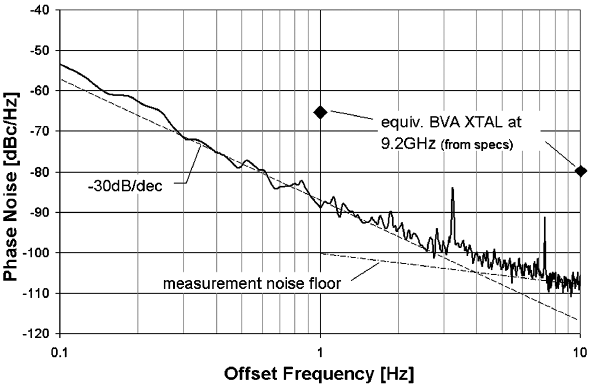

FIG. 8: *Phase noise of two conventional X-band cryogenic sapphire oscillators; the diamonds provides a comparison (referenced to 9.2 GHz) with a state-of-the art quartz oscillator*

It is interesting to speculate what the fundamental and/or practical limits on the WhiGMO's phase noise might be. Looking back to equation 1, one realizes that $S_\psi(f)$ is the phase noise of the distributed maser amplification process within the cryogenic sapphire crystal. Though the effective noise temperature (i.e. the level of *white noise*) of solid-state (predominantly ruby) maser amplifiers was investigated[12] in the late 1950's and early 1960s, the *flicker noise* associated with solid-maser action, which is a "quantum" gain process, fundamentally different from the gain supplied with rf/microwave transistors, is very poorly understood. Since the masering volume of the sapphire ring is relatively large (many cm^3), and it is known that flicker noise can be suppressed through spatial averaging (e.g. ganging several low-noise amplifiers up in parallel, or using a voluminous power amplifier in place of a smaller "low-noise" amplifier that exhibits greater flicker), one might speculate that the flicker noise associated with the WhiGMO's maser action (as opposite to more "technical" noise contributions) could be extremely, perhaps immeasurably, low.

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