INTERCOMPARISON OF QUASI-OPTICAL AND WAVEGUIDE TECHNIQUES FOR THE MEASUREMENT OF THE COMPLEX DIELECTRIC CONSTANTS OF SOLID MATERIALS

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An intercomparison has been made of quasi-optical and waveguide-based techniques for the measurement of the complex dielectric constants of solid materials in the frequency range 75 to 110 GHz. Plane parallel-sided samples are used in both systems, acting as Fabry-Perot etalons. Models based on ABCD matrices, and assuming no dispersion, are fitted to the interference patterns measured in transmission and reflection to determine the real and imaginary parts of the dielectric constant. An additional waveguide-based system is used to obtain further data in the frequency range 26.5 to 40 GHz. Good agreement has been found between the results from the quasi-optical and waveguide-based systems, and also between our results and published dielectric constant data. Typically, systematic errors in the refractive index results are less than 1%, although errors in the absorption index are around 15%.

I. INTRODUCTION

Many systems for the measurement of complex dielectric constants have been previously described. Afsar *et al.* [1] describes most of the available techniques, also providing an intercomparison of the results obtained using the various methods, and detailing the most applicable technique for the frequency range of interest. A further intercomparison for some common low loss dielectric materials is given by Birch *et al.* [2].

II. THEORY

The complex dielectric constant of a material contains information about its permittivity, ε', and loss, ε''. Thus the complex dielectric constant is expressed as

$$\hat{\varepsilon} = \varepsilon' + i\varepsilon'' \tag{1}$$

where i is the square root of -1.

It is however more common to express these values as refractive index, n, and loss tangent, $\tan\delta$. These quantities are calculated from $\hat{\varepsilon}$.

$$\tan\delta = \frac{\varepsilon''}{\varepsilon'} \qquad (2)$$

The complex form of the refractive index, $\hat{n}$, is defined as

$$\hat{n} = n + ik \qquad (3)$$

where n is the refractive index and k is the absorption index. Since $\hat{\varepsilon} = \hat{n}^2$ for nonmagnetic materials [1], equations (1) and (3) are related by

$$\varepsilon' = n^2 - k^2 \qquad (4)$$
$$\varepsilon'' = 2nk \qquad (5)$$

The above are the general relations, but for low loss materials they can be approximated to give

$$\varepsilon' = n^2 \qquad (6)$$
$$\tan\delta = \frac{2k}{n} \qquad (7)$$

Finally, the power absorption coefficient, α is defined as

$$\alpha = \frac{4\pi\nu k}{c} \qquad (8)$$

where ν is the frequency and c is the speed of light in vacuum.

The work by Seeger [3] using a WG system to measure the refractive index of high-density polyethylene (HDPE) was used as a basis for our initial model of our waveguide-based system. However, the method used by Seeger cannot return any information about the loss term of the complex dielectric constant, ε''.

In both of our measurement systems, as in that used by Seeger, the sample acts as a Fabry-Perot etalon, of length, l. Whereas Seeger only considered the real part of the dielectric constant, ε', we use the complex dielectric constant, $\hat{\varepsilon}$, hence power incident upon the sample is either transmitted, reflected or absorbed, as shown in Fig. 1.

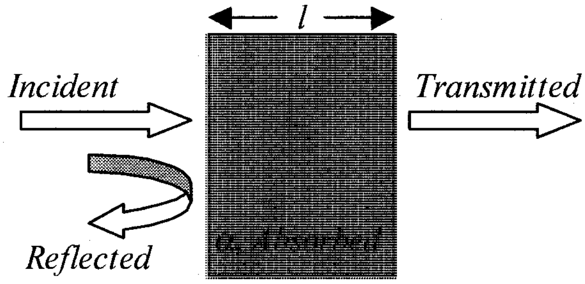

Fig. 1. Incident power is transmitted, reflected or absorbed by the sample

In order to analyse our results and measure both real and complex parts of the dielectric constant, an ABCD matrix method is used. The general form of the equations used for modelling both the WG and the QO systems is the same, the differences being in the calculation of the phase constants and the impedances of the samples within the systems.

The ABCD matrix for a generalised lossy transmission line is [4]

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh \gamma l & Z_0 \sinh \gamma l \\ \dfrac{\sinh \gamma l}{Z_0} & \cosh \gamma l \end{bmatrix} \tag{9}$$

where $\gamma = \alpha + i\beta$, is the propagation constant, with α as the power absorption coefficient and β as the phase constant. The length and characteristic impedance of the transmission line are respectively l and Z_0.

In the QO case, Z_0 is simply $1/\sqrt{\hat{\varepsilon}}$, α is given by (8) and

$$\beta_{QO} = \frac{2\pi\nu\sqrt{\hat{\varepsilon}}}{c} \tag{10}$$

In the WG case, α is again given by (8). The equations for Z_0 and β must include additional terms to account for the cut-off frequency of the waveguide cell in which the sample is placed.

$$\beta_{WG} = \frac{2\pi\nu\sqrt{\hat{\varepsilon} - \left(\dfrac{c}{2a\nu}\right)^2}}{c} \tag{11}$$

When considering the characteristic impedance, Z_0, of the sample inside the waveguide cell, it is necessary to consider the impedances of both the loaded and unloaded sections of waveguide. Goodwin and Moss [5] state that the impedances of two sections of dielectric-filled waveguide of the same width and height, but with differing dielectrics are

$$Z_1 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{1}{\sqrt{\left(\varepsilon_1 - \left(\frac{c}{2av}\right)^2\right)}} \tag{12}$$

$$Z_2 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{1}{\sqrt{\left(\varepsilon_2 - \left(\frac{c}{2av}\right)^2\right)}} \tag{13}$$

If Z_1 is taken to represent the section of unloaded waveguide, then $\varepsilon_1 = 1$ and therefore Z_2 represents the section of waveguide containing the sample with $\varepsilon_2 = \hat{\varepsilon}$. Combining (12) and (13) gives

$$Z_{0WG} = \frac{Z_2}{Z_1} = \frac{\sqrt{\left(1 - \left(\frac{c}{2av}\right)^2\right)}}{\sqrt{\left(\hat{\varepsilon} - \left(\frac{c}{2av}\right)^2\right)}} \tag{14}$$

For both the QO and WG cases, the resulting ABCD matrices are evaluated, with the transmitted and reflected powers being given by

$$\text{Trans} = \frac{2}{A+B+C+D} \tag{15}$$

$$\text{Refl} = \frac{A+B-C-D}{A+B+C+D} \tag{16}$$

These plots of transmitted and reflected power versus frequency are then fitted to measured data to determine the real and complex parts of the dielectric constant.

It should be noted that various assumptions are made in our model. The first assumption is that the materials being measured are of low enough loss that the approximations made by using (6) and (7) are valid. This is generally the case as long as tan $\delta \leq 500 \times 10^{-4}$. The other assumption is that the dielectric materials are non-dispersive. Although this is not always the case, for many common dielectrics the dispersion across the frequency range of interest is fairly small. It is also readily apparent from the results if a material is strongly dispersive as the periodicity of the transmission and reflection interferograms changes across the frequency band, and so moves out of phase with the model.

III. Experimental Arrangement

The waveguide-based systems, operating in the frequency ranges 26.5 to 40 GHz and 75 to 110 GHz, use split block waveguide cells to hold the samples. The split blocks comprise a base, with a channel of correct waveguide dimensions in which the sample is placed and a top, which is bolted to the base to form the roof of the waveguide. The ends of the block are machined with the flange pattern appropriate to the waveguide size used to allow the split block to be connected to the rest of the system. Fig. 2 shows a schematic diagram of the WG systems used for measurements in transmission and reflection.

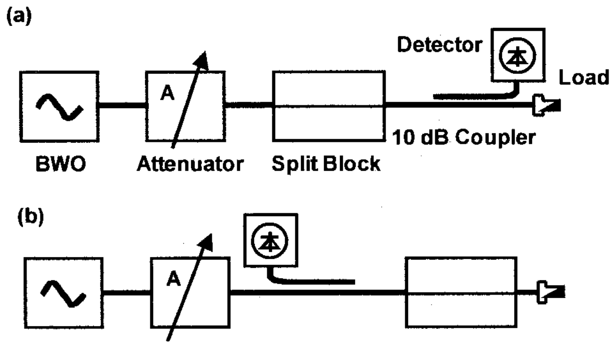

Fig. 2. Schematic diagrams of WG systems used for (a) transmission and (b) reflection measurements

The QO system only operates in the frequency range 75 to 110 GHz. Samples for the QO system take the form of 100mm by 100mm tiles of various thicknesses that are positioned normal to the beam. Fig. 3 shows the arrangements used for transmission and reflection measurements.

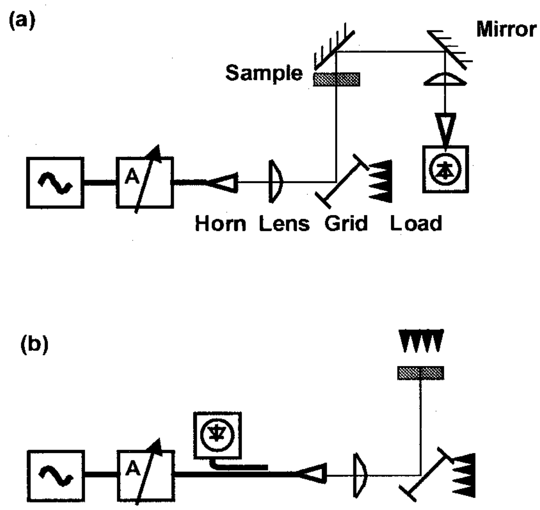

Fig. 3. Schematic diagrams of QO systems used for (a) transmission and (b) reflection measurements

In both the WG and QO systems it is necessary to establish a reference power level, without a sample, before performing a data run. This reference run establishes the power level supplied by the backward wave oscillator (BWO) at each incremental frequency across the measurement band and so quantifies the effect of introducing a sample into the system. In the case of the transmission measurements, for both the WG and QO systems, reference runs are taken with the either split block or the QO sample holder in position, but with no sample. For reflection measurements in the WG system the reference is taken with the waveguide cell in position, but with no sample, and the load being replaced with a short. In the QO system the reflection reference is taken by replacing the sample with a plane mirror to reflect back all of the incident power.

The 75 to 110 GHz systems use an ELVA-1 G4-141dM (GPIB) BWO, a Boonton 4220 power meter and a Boonton 51407 (4W) power sensor. The 26.5 to 40 GHz system uses a Marconi Instruments Type 6600A Microwave Sweep Oscillator with a Hewlett Packard 438A power meter and HP R8486D power sensor. These systems are all controlled using an Acorn Archimedes A540 computer via GPIB. The frequency sweeping of both systems under computer control was calibrated using an EIP model 578 source locking microwave counter.

IV. DATA ANALYSIS

The reference run data is subtracted from the measured data for each sample to reveal the characteristic Fabry-Perot interference patterns in transmission and reflection. Data analysis is performed using Mathcad 2001i[†] to model the theoretical propagation equations and plot these against the measured interferograms. A least squares error minimisation is used to determine the values of ε' and ε'' which provide the best fit of the theoretical curves to the measured data.

V. EXPERIMENTAL RESULTS

Fig. 4 shows a typical data set and numerical fit for a WG sample in the frequency range 75 to 110 GHz. In this case the sample is of Rexolite®1422[‡] and the fit yields a value of $\varepsilon' = 2.505$, and $\tan \delta = 8.0 \times 10^{-4}$.

Fig. 5 shows a data set and numerical fit for a QO sample in the same frequency range. Again this is for a Rexolite sample, with the fit giving $\varepsilon' = 2.53$, and $\tan \delta = 8.1 \times 10^{-4}$.

Fig. 6 shows a data set and numerical fit for a Rexolite WG sample in the frequency range 26.5 to 40 GHz. In this case the fit gives a value of $\varepsilon' = 2.48$, and $\tan \delta = 8.1 \times 10^{-4}$.

† http://www.mathsoft.com/

‡ Rexolite® 1422 is a registered trademark of C-Lec Plastics, Inc.

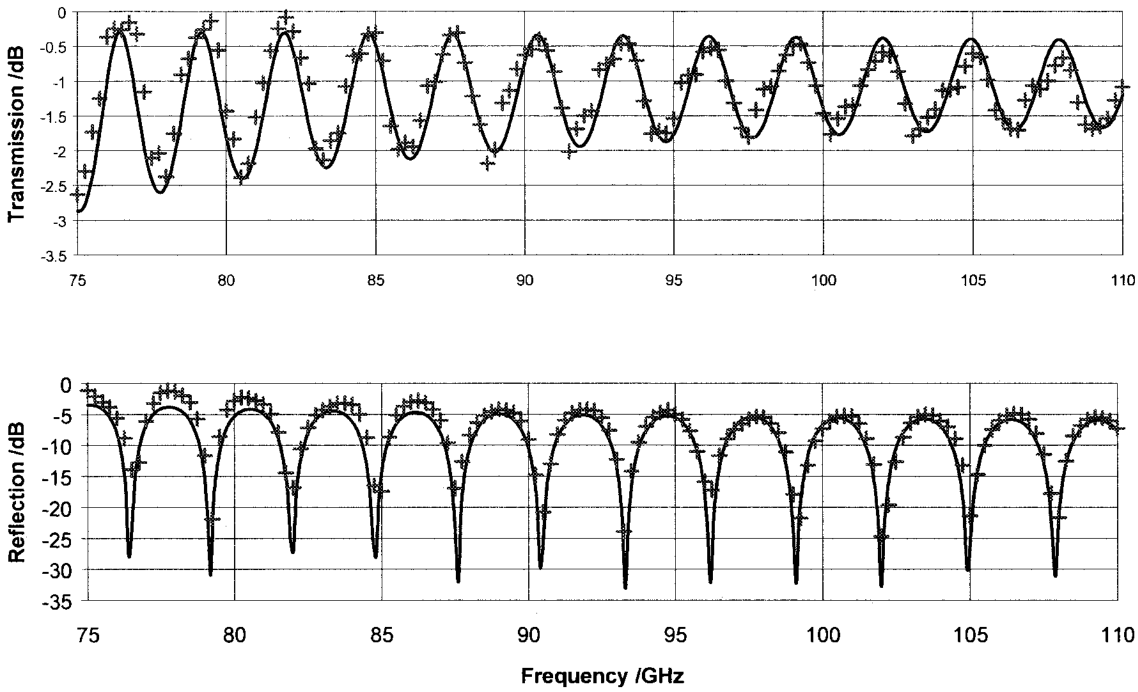

Fig. 4. Data and numerical fit for transmission and reflection for a 30.04mm Rexolite waveguide sample

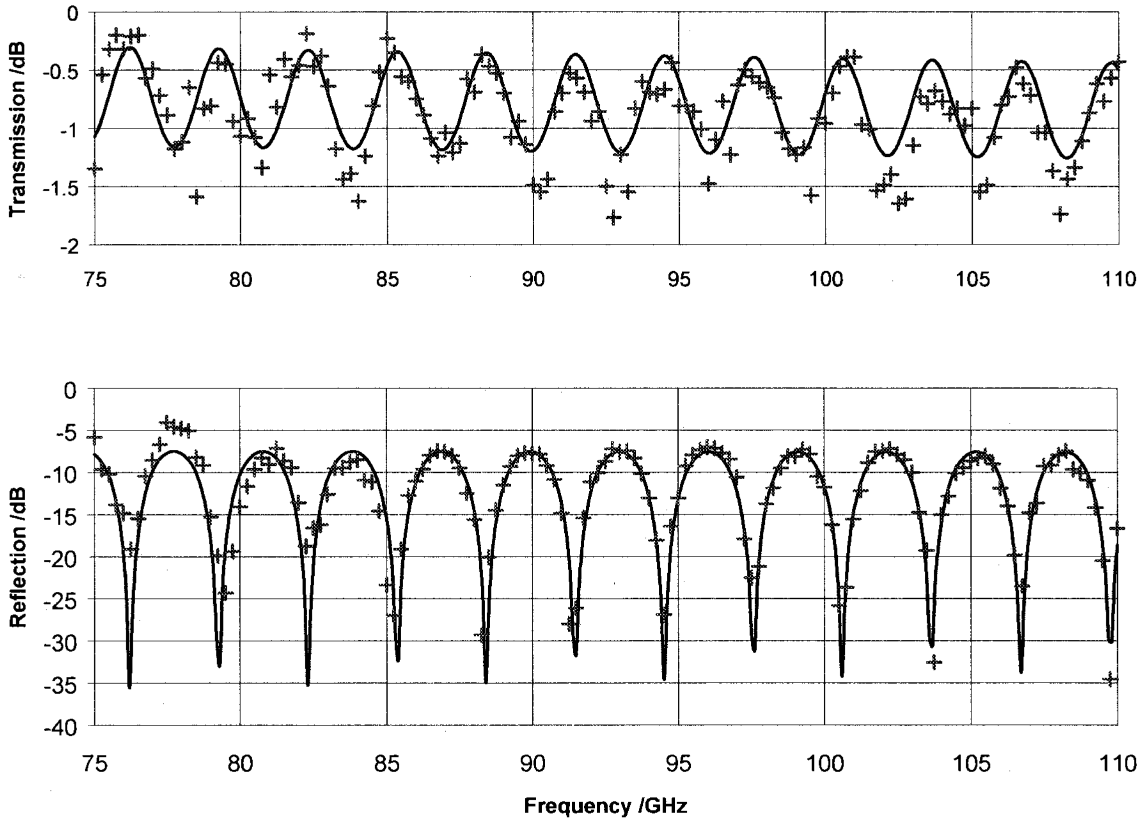

Fig. 5. Data and numerical fit for transmission and reflection for a 30.91mm Rexolite quasi-optical sample

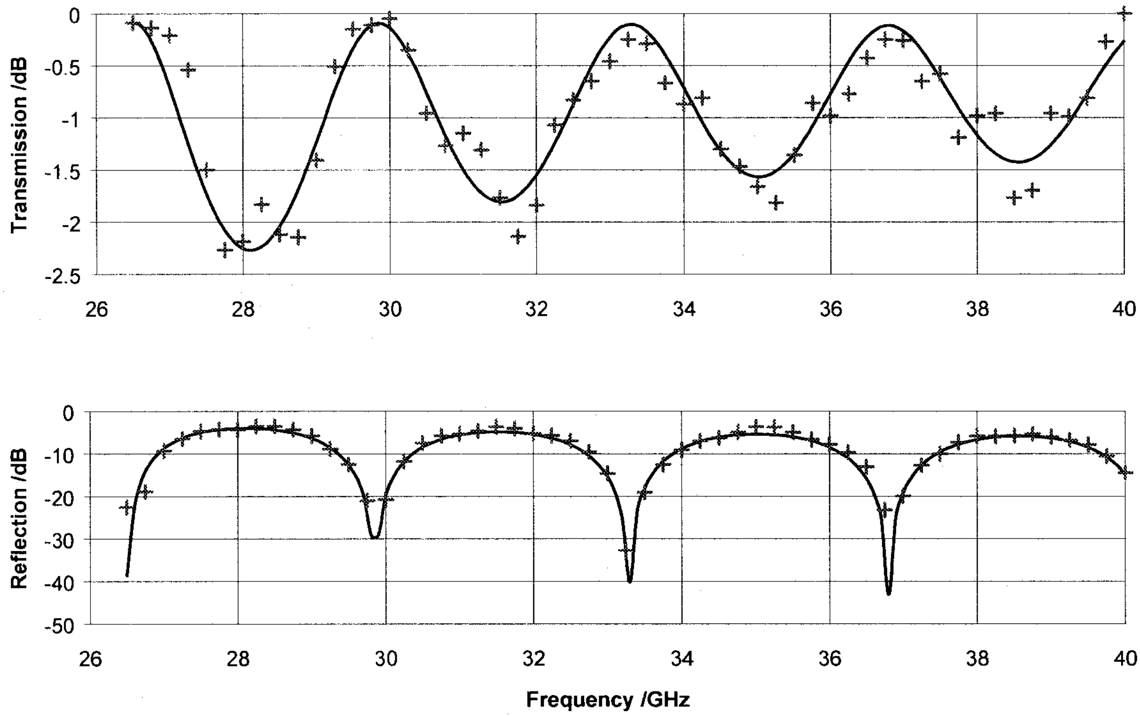

Fig. 6. Data and numerical fit for transmission and reflection for a 25.02mm Rexolite waveguide sample

Table I shows a summary of results for five dielectric materials tested in the frequency range 75 to 110 GHz. The results presented are the average values of ε' and $\tan\delta$ measured for three lengths of WG samples and of two thicknesses of QO samples. For comparison, data from Lamb [6] is also included.

	WG Data		QO Data		Data from Lamb [6]	
Material	ε'	$\tan\delta \times 10^4$	ε'	$\tan\delta \times 10^4$	ε'	$\tan\delta \times 10^4$
Fluorosint® 500†	3.30 ± 0.03	24 ± 2	3.58 ± 0.03	27 ± 5	3.54	17
HDPE‡	2.28 ± 0.02	8.8 ± 1	2.29 ± 0.02	8.7 ± 1	2.32	3.1
PMMA§	2.58 ± 0.02	112 ± 10	2.61 ± 0.02	115 ± 10	2.58 – 2.61	20 – 90
Rexolite® 1422	2.51 ± 0.02	8.0 ± 1	2.53 ± 0.01	9.4 ± 1	2.50 – 2.58	2.6 – 10
TPX®¥	2.11 ± 0.02	9.5 ± 1	2.13 ± 0.01	9.4 ± 1	2.12 – 2.15	6 – 10

Table I. Summary of measurements with comparison to previously published data for 75 to 110 GHz

† Fluorosint® 500 is a registered trademark of Quadrant Engineering Plastic Products

‡ HDPE is High Density Polyethylene

§ PMMA is Polymethyl Methacrylate, also known as acrylic

¥ TPX® is a registered trademark of Mitsui Petrochemical Industries, Ltd.

Table II shows a summary of results for two dielectric materials tested in the frequency range 26.5 to 40 GHz. The results presented are the average values of ε' and $\tan\delta$ measured for three lengths of WG samples. For comparison, data from Birch *et al.* [7] is shown for Rexolite for the frequency range 30 – 40 GHz is also included. Unfortunately the authors have been unable to find a reference for Fluorosint measurements in this frequency range.

	WG Data		Data from Birch *et al.* [7]	
Material	ε'	$\tan\delta \times 10^4$	ε'	$\tan\delta \times 10^4$
Fluorosint® 500	3.52 ± 0.02	22.7 ± 2	–	–
Rexolite® 1422	2.49 ± 0.01	8.2 ± 1	2.53	5.8

Table II. Summary of measurements for 26.5 to 40 GHz with comparison to previously published data

VI. Summary

For all of the materials tested, both the QO and WG samples were made from the same piece of dielectric, ensuring consistency between samples under test for both techniques.

The results show generally good agreement between the values obtained using the WG and QO systems. There is also good agreement between our results and those previously collated and published by Lamb [6].

While there is good agreement between our 75 to 110 GHz QO data, our 26.5 to 40 GHz WG data and the data published by Lamb [6] for ε' for Fluorosint (3.58, 3.52 & 3.54), our 75 to 110 GHz WG result differs greatly (3.30). We do not currently know why this difference should arise, although this result is consistently repeated for our Fluorosint WG samples.

It is our intention to cross-calibrate further our WG and QO systems by performing DFTS measurements on our QO samples. We also intend to improve our system for making QO transmission measurements by suppressing standing waves, which are visible in Fig. 5.

Acknowledgment

The authors would like to thank Mr. Dave Steven and Mr. Andy Barman of the St Andrews Physics Department Mechanical Workshop for making the waveguide cells and for the preparation of the samples used in this experiment. We would also like to thank the Engineering and Physical Sciences Research Council for their funding of this Ph.D. project.

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