HIGH ORDER MODULATION
SHAPED TO WORK WITH RADIO IMPERFECTIONS

Karl Martin Gjertsen[1]
Nera Networks AS, P.O. Box 7090
N-5020 Bergen, Norway

ABSTRACT

A novel layout of constellations has been conceived, promising to allow large symbol alphabets to be used for radios with inferior equipment noise. A number of constructions have been analysed, showing behaviour as expected.

I. INTRODUCTION

Radios for high spectral efficiency have normally been using QAM-like constellations. At the cost of high quality hardware, it has been possible to achieve error-free transmission with as many as 1024 point constellations. Every symbol then carries 10 bits of information. This work is motivated by the potential for cost reductions if relaxed hardware requirements can be tolerated. High-order modulation, defining constellations with many points, means that the different symbols may have very different signal amplitudes. Impairments that have a multiplicative effect on the signal, like nonlinearity and phase noise, become more prominent with high order modulation. This work addresses the potential to shape a constellation with increased robustness to such effects, primarily focussing on the phase noise part.

Recently, sophisticated modulation schemes have gained extended interest. For example the new DVB-S2 system allows a number of different constellations to be used and varied with varying coding schemes [1]. This shows that the concept of a fully programmable software radio, which adapts to the channel conditions, is no longer only a theoretical concept but will actually be implemented. For DVB-S2 the reason for choosing such advanced schemes has not been phase noise, but mainly due to the improved performance with respect to non-linear amplifiers. After describing the properties of the proposed constellation with respect to phase noise, we will briefly indicate some of the potential advantages concerning nonlinearity. In [2] is also given designs of constellations taking phase noise into consideration, but the results focus on smaller constellations and uncoded systems.

II. CONSTRUCTION OF THE CONSTELLATION

The signal path from modulator to receiver contains a number of noise sources in addition to the additive white noise in the receiver, defining the reception threshold. A model incorporating multiple noise sources is developed, being the foundation for the construction of constellations taking the total noise into consideration. The resulting constellations are named Noise Dependant Constellations, abbreviated NDC.

A. Noise model

Following the signal path through a radio, a number of noise sources are encountered, creating random offsets from the transmitted signal. A generalized signal path may look like shown in Fig. 1.

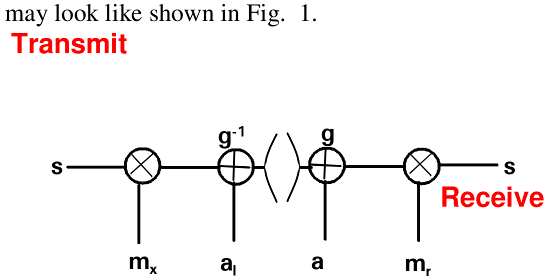

Fig. 1: Signal path with noise sources

Fundamentally, we find that the transformation of a signal through a radio link permits noise to be introduced additively and multiplicatively. The main contribution to additive noise is normally the low-noise amplifier, while phase noise from oscillators is considered to be the main source for multiplicative noise. In the figure, we vaguely indicate a transmit and a receive part, even if there is no reason to make this distinction.

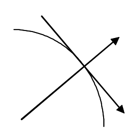

Fig. 2: Cartesian decompostion of noise

Variations caused by phase noise cause a rotation of the signal vector. If they are small enough to be described by a Gaussian distribution along an axis perpendicular to the signal vector, it is possible to give

[1] The work is performed in cooperation with Bjarne Risløw, Dagrun Røyrvik and Helge Coward at Nera Research

a good approximation of the noise distribution in the received signal.
Accumulating received signals generated with the same amplitude and phase, the expected noise distribution can be described in a Cartesian reference frame as shown in Fig. 2. The components along the two axes can be described as independent Gaussian distributions where the variance is the sum of a constant term and a term proportional to the signal power. As a consequence, we find that contours of constant probability density are elliptic contours around the origin in the established coordinate system. These contours serve as good guidelines for defining constellations promising a given error rate at a given noise level.

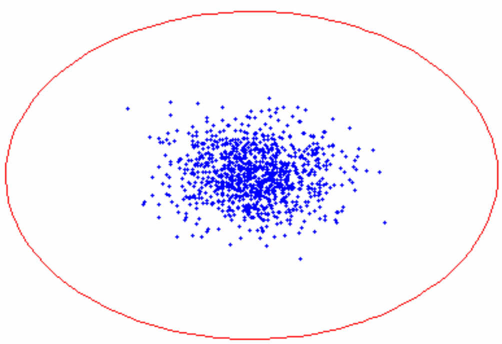

Fig. 3: Illustration of a noise scenario

A constellation is the arrangement of the set of signal vectors (symbols) defined to represent valid data values. In the receiver, a decision process will make a best estimate of which signal vector was transmitted. If every signal vector is associated with a region where it is likely to find the incoming value in the receiver, a constellation can be built by arranging the signals such that they have non-overlapping regions, called decision regions.

B. The construction of suitable constellations

The existence of elliptic contours makes it possible to envision a simple construction method for the constellations. As we want the set of regions to cover all possible values that can be received, we define decision regions by toroidal segments containing elliptic boundaries at a chosen probability level around each symbol point The probability of receiving samples of the symbol outside the boundary roughly scales with the probability level at the boundary, making it relatively simple to design for a desired error level. The toroidal shape suits situations with high phase noise, making the resulting constellations more robust than the assumptions in the analysis might indicate.
Arranging toroidal segments invite to describing the constellation in polar coordinates, giving very simple decision rules. Symbols are identified by amplitude intervals and corresponding phase intervals.
To find the decision regions, we start finding the ellipses giving the probability level and enclose them by the smallest possible toroids, then extending the angular extension to arrive at an even spacing around a circle at every amplitude level.

This construction method defines a ring structure allowing a number of symbols per ring in the constellation plane, and a well defined radial distance from ring to ring. As radios normally are limited by the peak amplitude available, we start constructing the constellations from outside. There is an option whether to place a symbol at the origin if there is room for it. We have chosen not to do so. In every ring there is an option to rotate the collection of symbols an arbitrary angle. We have not investigated the potential of exploiting that degree of freedom, starting at 0 in all rings.
The character of this constellation type becomes visible once the additive noise does not dominate the noise picture. Fig. 4 illustrates the concept.

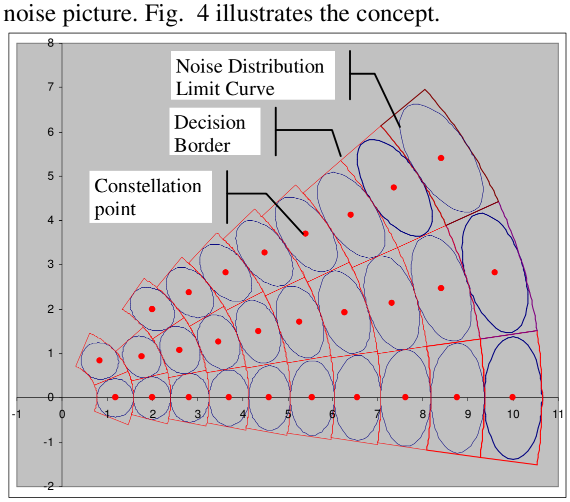

Fig. 4 Example constellation (sector)

C. *Scaling symbol alphabet with noise level*

The approach allows symbols to have a very high density if the noise is moderate (we do not assume significant multiplicative noise in the amplitude direction.). Phase noise parameters suitable for 128QAM constellations (7 bits/symbol) are tested and indicate that it should be possible to build constellations representing something like 14 bits/symbol if it is possible to have very low additive and multiplicative noise (SNR about 50 dB). This means that we might double the capacity of such a radio if the receive level is very good.

III. System Model

Simulation results of error rates are presented both with uncoded system and coded system, see Fig. 5. The uncoded system consists of a modulation, additive and multiplicative noise and demodulation. In our simulations we used maximum likelihood decoding and calculated the Euclidian distance from the received signal to each possible symbol in order to determine which symbol values had been received.

We also analysed a coded system based on Product Accumulate (PA) Codes, [3]. This FEC scheme can be decoded with a very simple iterative decoder and has quite good performance. The decoder requires soft decision values, so we need a good bit-to-symbol mapping.

We have used a 1st order decision directed PLL for phase tracking. The phase error after the PLL can be approximated by a Tikhonov distribution,[4] For high signal to noise ratios, as will be the case for large constellations, the pdf will be very closely approximated by a Gaussian distribution which makes our initial design criteria valid.

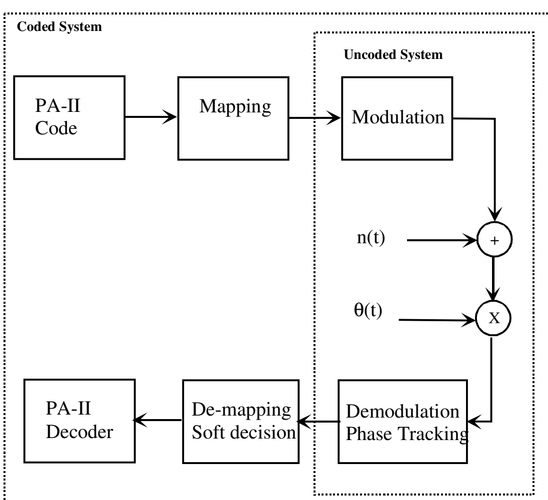

Fig. 5 System Model

IV. Soft decisions and bit mapping

In the receiver one must calculate the bit error rate resulting from the symbol error rate. This depends on the specific mapping of bits onto symbols. We used Gray[2] mapping in order to minimise the uncoded bit error probability.

The bit-to-symbol mapping is a crucial element of the coded system and how to get the best possible mapping for a generic constellation is still an open issue. We identified a mappig by using an algorithm searching to minimise the bit error rate at the selected noise level. The algorithm does not guarantee that we have found the best possible mapping, but serves to prove an obtainable performance level. Search for alternative algorithms could be a subject for further study.

It is important to emphasise that since we take into account the phase noise in our constellation design the mapping will depend on the phase noise level and can change significantly if the design is done for a different noise scenario.

V. SIMULATION RESULTS

A. *Constellation*

We demonstrate the performance by comparing a 128-QAM constellation and a 128 constellation designed with decision boundaries allowing 5º phase noise deviations.

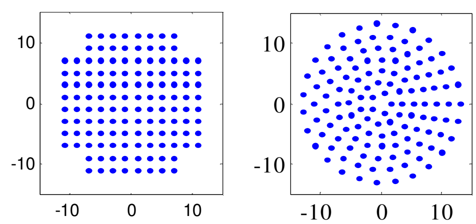

Fig. 6 An example of constellation with 128-QAM (left) and 128-NDC (right)

The robustness to phase noise can be seen from the response curve of a decision directed phase tracker, see Fig. 7. The 128-NDC clearly has a larger lock-in range than 128-QAM.

[2] In most cases Gray mapping minimize the coded bit error probability also. However, for these complex constellations it was not be possible to achieve a "true" Gray mapping.

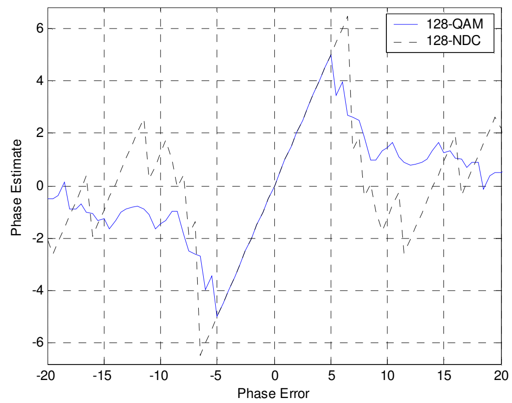

Fig. 7 S-Curve for the different constellations based on decision directed phase tracking

B. Uncoded System

Simulation results for the uncoded symbol error rate using the two constellations are shown in Fig. 8.

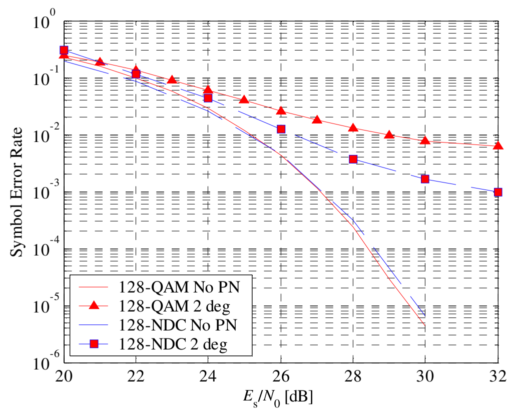

Fig. 8 Symbol error rate with and without phase noise

In this example we simulated without and with phase noise (White Gaussian phase noise with σ=2°). The performance without phase noise is more or less identical for the two constellations, but we can see a clear difference with phase noise where the NDC constellation is 2-4 dB better depending on the signal-to-noise level. Defining constellations for tolerating higher phase noise will normally trade degraded receiver threshold with better performance at high SNR levels.

In Fig. 9 is given results for a random walk phase noise model ($1/f^2$). In the simulations we have assumed a symbol rate of 10 Msps and a phase noise level of -67 dBc/Hz at 10kHz. With the NDC constellation we experienced that this constellation is more vulnerable to phase slips, and especially we see this with a random walk phase noise model. The reason for this can be seen from the phase detector response curves where the sign of phase error is correct up to 18° for the 128-QAM while for 128-NCD we get a sign error when the phase error is above 7.5°. In order to avoid the phase slips we included pilot symbols. In the simulations we inserted a pilot symbol for each 42th data symbol. This represents only 0.1 dB extra signal energy.

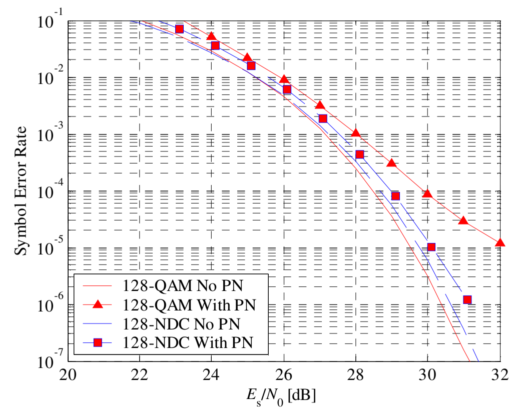

Fig. 9 Symbol error rate with and without phase noise. Random Walk Phase Noise (-67 dBc/Hz at 10kHz)

C. Coded System

Simulation results for a coded system is shown in Fig. 10. The phase noise is white Gaussian noise with σ=2°. The simulation were with the PA-II code with coderate, r=0.96, information block length, N=1048 and
8 iterations were used in the iterative decoder.

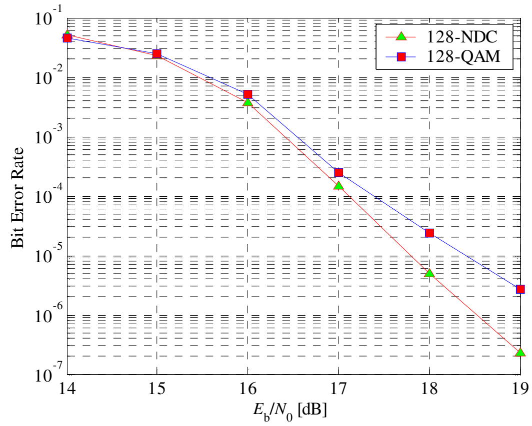

Fig. 10 Coded Bit error rate with phase noise

With the above FEC parameters an E_b/N_0 of 18 dB corresponds to E_s/N_0=26.3 dB we see that in the uncoded case one get 2-3 dB improvement which is reduced to a little less than 1 dB with coding. This may come from the imperfect Gray mapping. Even with this loss there is an improvement using the optimised constellation in competition with only a slightly degraded QAM constellation. If bigger symbol alphabets or higher phase noise was introduced, we expect to find NDC constellations giving good performance above a threshold value, while a QAM constellation would not work at any level.

VI. CIRCULAR CONSTELLATIONS AND NONLINEARITY

In the first place, the circular constellations have smaller peak-to-average and will be less degraded when the signal passes through a non-linear amplifier, allowing higher average power with the same amplifier, still satisfying spectrum requirements. If high symbol alphabets are used, the tolerance to nonlinearity for error-free reception is reduced. We assume that the nonlinearity is a memory-less function of symbol amplitude, introduced at a point where virtually no amplitude noise is present.

This allows a different kind of correction scheme. At the transmit side, it becomes essential to maintain the amplitude separation between the symbol levels. This can for instance be done by increased amplitude separation at high power level. At the receive side, one knows that the phase references at each level will be shifted. These phase reference shifts may be estimated by the receiver, leading to full-quality reception without very demanding linearization schemes.

VII. REFERENCES

[1] Digital Video Broadcasting (DVB) Second generation framing structure, channel coding and modulation systems for Broadcasting, Interactive Services, News Gathering and other broadband satellite applications, *ETSI draft Specification*, DVBS2-74r13

[2] G.J. Foschini, R.D.Gitlin and S.B. Weinstein: “On the Selection of Two-Dimensional Signal Constellation in the Presence of Phase Jitter and Gaussian Noise*,” The Bell system Technical Journal,* Vol. 52, No.6, July-August, 1973

[3] Li, J., Narayanan, K.R andGeorghiades, C.N, ”Product Accumulate Codes: A Class of Codes With Near-Capacity Performance and Low Decoding Complexity,” *Information Theory, IEEE Transactions on* ,Volume: 50 , Issue: 1 , Jan. 2004

[4] Proakis, J. G. "Digital Communications, Second Edition ", McGrawHill 1989.